1. Suppose the entire region below the plane $z = 0$ is filled with a uniform dielectric material with dielectric constant κ and a point charge Q is located at $(0,0,d)$.
   (a) Calculate the induced surface bound charge density σ_b on the plane $z = 0$ as a function of distance $s = (x^2+y^2)^{1/2}$ from the origin. (Hint: The z-component of the electric field just below $z = 0$ due to σ_b only is $-\sigma_b/2\epsilon_o$.) (10 points)
   (b) Calculate the z-component E_z of the electric field at $z = 0$ and just inside the dielectric as a function of s. (5 points)
   (c) The electric field inside the dielectric is the same as if Q is changed to Q' and the whole space is occupied by the dielectric. Find out Q'. (5 points)

2. Consider the magnetic field produced by a thin rod of radius R and length $2L$ with the rod center taken as the origin and the z-axis along the rod.
   (a) If the rod has a uniform surface charge density σ and spins around z-axis at an angular velocity $\vec{\omega}$ pointing $+z$-axis, calculate the magnetic induction $\vec{\mathbf{B}}$ at $(d,0,0)$, where $d >> R$. (Hint: The resultant $\vec{\mathbf{B}}$ has z-component only. Also $\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{1}{r^3}[3(\vec{\mathbf{m}}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \vec{\mathbf{m}}]$ from a magnetic dipole $\vec{\mathbf{m}}$.) (10 points)
   (b) Find out $\vec{\mathbf{B}}$ at the same location if the rod is at rest with no surface charge and has a uniform magnetization $\vec{\mathbf{M}}$ pointing $+z$-axis. (5 points)

3. Consider the propagation of electromagnetic waves in a conductor with $\vec{\mathbf{D}} = \epsilon\vec{\mathbf{E}}$, $\vec{\mathbf{B}} = \mu\vec{\mathbf{H}}$, and $\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$, where ϵ, μ, and σ depend on frequency only.
   (a) Derive the dispersion relation between the wave number k and angular frequency ω from Maxwell's equations. (Hint: Take free charge density $\rho_f = 0$.) (5 points)
   (b) Obtain the skin depth δ, i.e. the distance that an electromagnetic wave travels with its amplitude reduced by a factor of $1/e$, for a good conductor (make some appropriate assumptions). (3 points)
   (c) Suppose the number of free electrons per unit volume is N and a damping factor $1/\tau$ exists for the average velocity $\vec{\mathbf{v}}$ of the electrons. Write down the equation of motion for $\vec{\mathbf{v}}$ (ignore the influence of magnetic field) and get the complex σ. (4 points)
   (d) Obtain the critical angular frequency ω_p that the waves with frequencies higher than ω_p will propagate without attenuation and explain the reason. (Hint: $\omega\tau >> 1$.) (3 points)

4. $E = E_0(\sin\theta/\gamma)\,[\cos(kr - wt) - (1/kr)\sin(kr - wt)]\,\hat{\phi}$, where $w/k = c$

(a) Show that E obeys all four of Maxwell's equations, in vacuum, and find the associated magnetic field. (10 points)

(b) Calculate the Poynting vector, Average S over a full cycle to get the intensity vector I. (10 points)

(c) Integrate I over a sphere of radius R ($\oint \mathbf{I}\cdot d\mathbf{a}$) to get the total (average) power radiated. (5 points)

5. Assuming negligible damping ($v_j = 0$), calculate the group velocits ($v_g = dw/dk_+$) of the waves described by

$\vec{E}(x,t) = E_0 e^{-k_- x} e^{i(k_+ x - wt)}$ (5 points)

and $k = w/c\,[\,1 + (nq^2/2m\epsilon_0)\sum_j f_j/(w_j^2 - w^2) - i\gamma_j w]$

show that $v_g < c$, even when $v > c$ (5 points)

6. Explain (1) Why the sky is blue (daytime) (5 points)
(2) Why the Ocean's color is light blue or deep blue depends on the depth of the water. (10 points)