1. Find the particular solution corresponding to the initial conditions,

$$x^2 y''' - 2y' = 0; \quad y(1) = 2, \quad y'(1) = y''(1) = 0$$

(10%)

2. Obtain the general solution by the method of elimination,

$$x' = \sin t - y$$
$$y' = -9x + 4$$

(10%)

3. Solve for x (t), on $0 \le t < \infty$

$$x'''' - x = \delta(t-1), \quad x(0) = x'(0) = x''(0) = x'''(0) = 0$$

(10%)

4. Determine the rank, nullity, number of linearly independent rows , and number of linearly independent columns for the given matrix.

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 1 \end{bmatrix}$$

(10%)

5. Evaluate the inverse matrix

$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

(10%)

6. (10 points)

Compute the eigenvalues of the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

7. (10 points)

u is a scalor field and ∇^2 is the Laplacian operator. <u>Evaluate</u> $\nabla^2 u$ for

$$u(x,y) = (x)\,(e^y)\,.$$

8. (10 points)

The curve C is defined parametrically by

$$x(\tau) = \cos(\tau), \quad y(\tau) = \sin(\tau), \quad \text{with} \quad 0 \le \tau \le \pi/2;$$

i.e., counterclockwise along the circle $x^2+y^2=1$ from $(0,1)$ to $(1,0)$. <u>Evaluate</u> the line integral

$$\int_C \left(3x^2 + 3y^2\right) ds.$$

9. (10 points)

C is the counterclockwise circle $|z| = 5$, which is a closed curve. Evaluate the complex integral

$$\oint_C \frac{e^z}{z-3}dz$$

10. (10 points)

Compute the Fourier series of the below periodic function $f(x)$, where $f(x)$ is given over one period as follows

$$f(x) = x \quad \text{for} \quad -\pi < x \leq \pi.$$