八十五學年度 化學工程學系 系（所） 甲組 組碩士班研究生入學考試

科目 工程數學 科號 1503 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

Problem 1 (20%)

Find a particular solution for each of the following ordinary differential equations:

(i) $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=x^3+x$ (5%)

(ii) $\frac{d^3y}{dx^3}+y=e^{2x}\cos 3x$ (5%)

(iii) $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-5y=6x^5$ (5%)

(iv) $\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+(x^2+2)y=0$ (5%)

Problem 2 (20%)

10% (a) Find the inverse and the determinant of $\mathbf{A}=\begin{bmatrix}2 & 4 & 3 & 2\\ 3 & 6 & 5 & 2\\ 2 & 5 & 2 & -3\\ 4 & 5 & 14 & 14\end{bmatrix}$

by the Gauss-Jordan elimination.

10% (b) Show that adj(adj $\mathbf{A}$)=$|\mathbf{A}|^{n-2}\cdot\mathbf{A}$, *if* $|\mathbf{A}|\neq 0$

($\mathbf{A}=[a_{ij}]$ be an n-square matrix and A_{ij} be the cofactor of a_{ij}; then by definition, adjoint ($\mathbf{A}$)=adj($\mathbf{A}$)=$[A_{ij}]$).

Problem 3 (20%)

Solve the following two partial differential equations

(a) $px\frac{\partial Z}{\partial x}-py\frac{\partial Z}{\partial y}=q\frac{\partial^2 Z}{\partial y^2}$

$x=0,\quad Z=Z_s$

$y=0,\quad Z=Z_w$

$y=\infty,\quad Z=Z_s$

where p and q are constants.

(b) $$\frac{\partial Z}{\partial t} = p\,\frac{\partial^2 Z}{\partial x^2}$$

$$t = 0, \quad Z = Z_0$$

$$x = 0, \quad \partial Z / \partial x = 0$$

$$x = 1, \quad -\partial Z / \partial x = qZ$$

where p and q are constants.

Problem 4 (20%)

Consider the rectangular (Cartesian) coordinates (x_1, x_2, x_3) and a general orthogonal coordinates (q_1, q_2, q_3). The two coordinates are connected through the following relations:

$$\begin{cases} x_1 = x_1(q_1, q_2, q_3) \\ x_2 = x_2(q_1, q_2, q_3) \\ x_3 = x_3(q_1, q_2, q_3) \end{cases}$$

(a) Give an expression for the position vector $\bar{r}$ in terms of the rectangular coordinates and their unit vectors $\left[\bar{\delta}_1, \bar{\delta}_2, \bar{\delta}_3\right]$. (3 %)

(b) Find $\dfrac{\partial \bar{\delta}_i}{\partial q_j}$ for $i = 1,3$ and $j = 1,3$. (3 %)

[Hint: how would $\bar{\delta}_i$ change its magnitude and direction along coordinate q_j?]

(c) Derive expressions for the unit vectors $\left[\vec{e}_1, \vec{e}_2, \vec{e}_3\right]$ of the general orthogonal coordinates.

[Hint: the unit vector of coordinate q_j is the unit tangenet vector to coordinate q_j.]

(10 %)

(d) Let (q_1, q_2, q_3) represent the elliptical cylindrical coordinates defined as

$$\begin{cases} x_1 = a\cosh q_1 \cos q_2 \ ; \quad q_1 \ge 0 \\ x_2 = a\sinh q_1 \sin q_2 \ ; \quad 0 \le q_2 \le 2\pi \\ x_3 = q_3 \ ; \quad -\infty \le q_3 \le \infty \end{cases}$$

where a is a constant. Express $[\bar{e}_1, \bar{e}_2, \bar{e}_3]$ in terms of (q_1, q_2, q_3) and $[\bar{\delta}_1, \bar{\delta}_2, \bar{\delta}_3]$. (4%)

Problem 5 (20%)

Let Fourier integral be denoted as the following:

$$f(t) = \int_{-\infty}^{\infty} g(\omega) e^{-i\omega t}\, d\omega$$

$$g(\omega) = \int_{-\infty}^{\infty} f(\tau) e^{-i\omega\tau}\, d\tau$$

(1) Find the Fourier integral of the following function (10%)

$$f(t) = \begin{cases} 1 & t^2 < 1 \\ 0 & t^2 > 1 \end{cases}$$

(2) Use your result in (1), find a particular integral of the following differential equation: (10%)

$$y'' + 3y' + 2y = f(t)$$

where $f(t)$ is denoted in (1).