國立清華大學命題紙

八十六學年度 化學工程學系 系（所） 甲 組碩士班研究生入學考試

科目 工程數學 科號 1603 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

Problem 1 (20%)

Solve the following two ordinary differential equations using Laplace Transform:

(i) $y''-6y'+9y=t^2e^{3t}$

subject to $y(0)=2,\ y(1)=\frac{25}{12}e^3$

(ii) $y''+16y=f(t)$

where $f(t)=\begin{cases}\cos 4t & 0\le t\le \pi\\ 0 & t>\pi\end{cases}$

and $y(0)=0,\ y'(0)=1$

Problem 2 (20%)

Solve $\frac{dy}{dx}=y-x-1+(x-y+2)^{-1}$

Problem 3 (20%)

a square matrix $\mathbf{A}=[a_{ij}]$ is called

Hermitian if $\overline{\mathbf{A}}^T=\mathbf{A}$, that is, $\bar{a}_{ji}=a_{ij}$

Skew-Hermitian if $\overline{\mathbf{A}}^T=-\mathbf{A}$, that is, $\bar{a}_{ji}=-a_{ij}$

unitary if $\overline{\mathbf{A}}^T=\mathbf{A}^{-1}$

where $\overline{\mathbf{A}}=[\bar{a}_{ij}]$ and $\bar{a}_{ij}$ is the complex conjugate of a_{ij}.

Show that

10% (a) The eigenvalues of a Hermitian matrix are real.

5% (b) The eigenvalues of a skew-Hermitian matrix are pure imaginary or zero

5% (c) The eigenvalues of a unitary matrix have an absolute value of 1.

Problem 4 (20%)

To solve the following partial differential equation

$$(1-x^2)\frac{\partial T}{\partial y} = \frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial T}{\partial x}\right) \qquad (1)$$

with the initial and boundary conditions:

(a) $x = 0, \quad T = \text{finite}$ (2a)

(b) $x = 1, \quad -\dfrac{\partial T}{\partial x} = 1$ (2b)

(c) $-y = \int_0^1 T(1-x^2)x\,dx$ (2c)

One may assume that the solution has the following form:

(i) $T = Ay + M(x)$ (3a)

(ii) $T = Ay \cdot M(x)$ (3b)

here A is a constant and M is a function of x.

which of (i) or (ii) is correct? and what should be the solution of (1) and (2)?

Problem 5 (20%)

(a) Given $\underline{F} = x\underline{i} + y\underline{j} + z\underline{k}$ and a region D bounded by the concentric spheres $x^2+y^2+z^2=a^2$ and $x^2+y^2+z^2=b^2$, b>a, find the outward flux $\iint_S (\underline{F}\cdot\underline{n})dS$ of the given $\underline{F}$. Here S is the bounding surfaces of D and $\underline{n}$ is the unit outward normal vector of S. (6%)

(b) Suppose that S is a smooth surface enclosing region D. Show that the volume of D is given as $\frac{1}{3}\iint_S \underline{r}\cdot\underline{n}dS$ where $\underline{r}$ is the position vector. (6%)

(c) Starting from the following variation of the divergence theorem: (8%)

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$$\iiint_V \left[\underline{\nabla} \cdot \underline{\underline{\tau}}\right] dV = \iint_S \left[\underline{n} \cdot \underline{\underline{\tau}}\right] dS .$$

Show that

$$\iiint_V \underline{\nabla}\phi dV = \iint_S \underline{n}\phi dS .$$

Here, S is a smooth surface enclosing region V, $\underline{\underline{\tau}}$ a second rank tensor, and ϕ a scalar field.