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Problem 1 (20%)

Flow in a Tube with a Sinusoidally Varying Cross-Section

Figure A shows a tube with a radius of the form $R(z) = R_0[1+\alpha\sin(\pi z/L)]$ where $|\alpha| < 1$ and R_0 is the mean radius. The pressure drop corresponding to the length of one full cycle, 2L, is denoted by $P_0 - P_{2L}$.

a. When $L >> R_0$ use simple approximation to develop an expression for the volume rate of flow Q for a given average pressure gradient $(P_0 - P_{2L})/2L$.

b. When $|\alpha| << 1$ show that the expression becomes approximately.

$$Q = \frac{\pi R_0^4 (P_0 - P_{2L})}{16\mu L[1+5\alpha^2]}$$

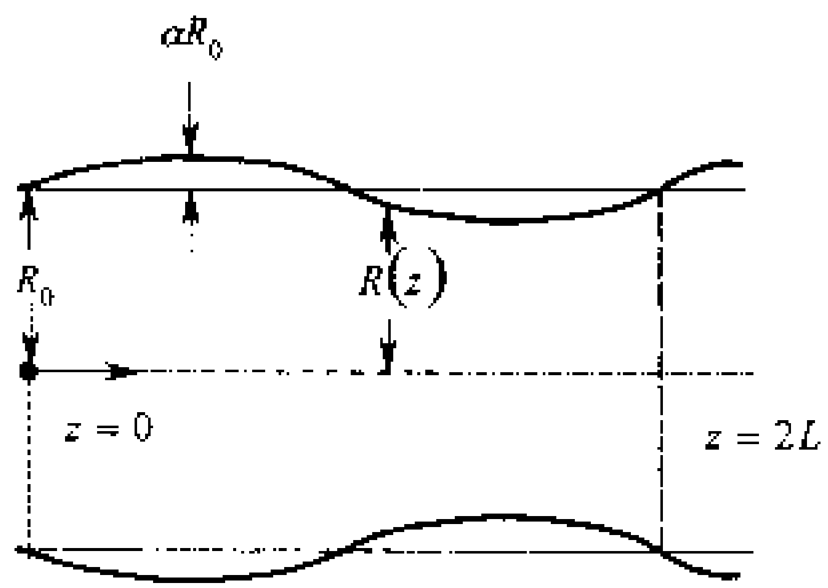


FIGURE A. Section of tube with a sinusodially varying radius in a cylindrical coordinate system

Problem 2 (20%)

Diffusion is occuring in the following two-phase solid system in which the two phases are immiscible. Solute is at a uniform concentration initially of C_I^0 in phase I and a uniform concentration of C_{II}^0 in phase II at time t=0. The solute is diffusing from phase I to II and it is assumed that Fick's second law holds in both phases. It is further assumed that there is equilibrium at the interface which can be described by the relation $C_{II} = mC_I$, where m is the distribution coefficient.

(8%) (a) Find the governing equations for C_I and C_{II} and the boundary conditions.

(12%) (b) Solve the equations obtained in (a).

Note that the error function is defined by

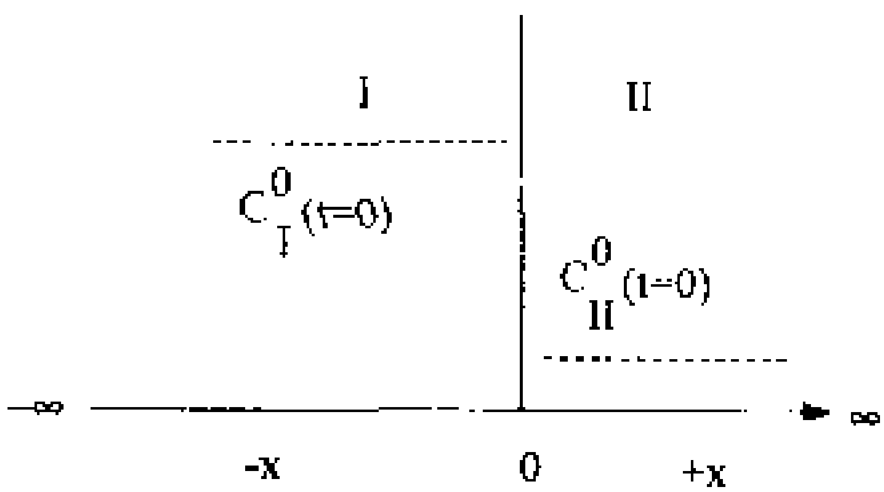
$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

$$\mathrm{erf}(\infty) = 1,\ \mathrm{erf}(0) = 0,\ \mathrm{erf}(-x) = -\mathrm{erf}(x)$$

$$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$$

The Laplace transform of $\mathrm{erfc}(\frac{a}{2\sqrt{t}})$ is

$$L[\mathrm{erfc}(\frac{a}{2\sqrt{t}})] = \frac{e^{-a\sqrt{s}}}{s}$$



Problem 3 (20%)

For a Newtonian fluid of constant pressure and constant thermal conductivity, k, the equation of thermal energy with a non-negligible viscous heating may be written as:

$$\rho \hat{C}_p \frac{DT}{Dt} = k\nabla^2 T + \mu\Phi_v,$$

where ρ is the fluid density, $\hat{C}_p$ the heat capacity per unit mass, μ the fluid viscosity, and Φ_v the dissipation function (note: $\mu\Phi_v = -\underline{\underline{\tau}}:\underline{\nabla}\underline{v}$).

(a) Non-dimensionalize the above equation by using the characteristic velocity, V, characteristic length, D, and characteristic temperature difference ($T_1 - T_0$) of the system, and show that the above equation becomes:

$$\frac{DT^*}{Dt^*} = \frac{1}{\Pr \mathrm{Re}}\nabla^{*2}T^* + \frac{Br}{\Pr \mathrm{Re}}\Phi_v^*. \qquad \text{(9 points)}$$

Here, the asterisk signifies the dimensionless nature of the variable and Pr, Re, and Br are the Prandtl number, Reynolds number, and Brinkman number, respectively.

(b) Give physical meanings for the three dimensionless groups. (9 points)

(c) Give the SI unit for Φ_v. (2 points)

Problem 4 (20%)

A porous solid is dried in a dryer under constant drying conditions, It takes 6 hr to reduce the moisture content from 30 to 10%. The critical moisture content was found to be 16% and the equilibrium moisture content is 2%. All moisture contents are on the dry basis (Kg water/Kg dry solid). Assuming that the rate of drying during the falling-rate period is proportional to the free-moisture content, how long should it take to dry the same solid from 35 to 6% under the same conditions?

Problem 5 (20%)

A plant must distill a mixture containing 75 mole percent methanol and 25 percent water. The overhead product is to contain 99.99 mole percent methanol and the bottom product 0.002 mole percent. The feed is cold, and for each mole of feed 0.15 mol of vapor is condensed at the feed plate. The reflux ratio at the top of the column is 1.4, and the reflux is at its bubble point. Calculate (*a*) the minimum number of plates; (*b*) the minimum reflux ratio; (*c*) the number of plates using a total condenser and a reboiler, assuming an average Murphree plate efficiency of 72 percent; (*d*) the number of plates using a reboiler and a partial condenser operating with the reflux in equilibrium with the vapor going to a final condenser. Equilibrium data are given in following Table.

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TABLE

Equilibrium data for methanol-water

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y	0.417	0.579	0.669	0.729	0.780	0.825	0.871	0.915	0.959	1.0