Problem 1 [(a)10%, (b)10%]

A cylindrical rod with a radius r_0 is rotating with an angular velocity Ω at steady state as shown in the following figure.

(a) Determine the velocity distribution for the tangential laminar flow of an incompressible fluid in the vicinity of the rod at steady state. End effects may be neglected.

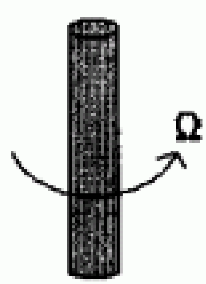
(b) At time t = 0, the rod is stopped rotating. Define the governing equation, initial condition, and boundary conditions to obtain the velocity distribution in the vicinity of the rod. You do not need to derive the velocity distribution.

The equation of motion in cylindrical coordinates are given below.

r-component
$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]+\rho g_r$$

θ-component
$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]+\rho g_\theta$$

z-component
$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}$$
$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$



Problem 2 [(a) 2%, (b) 3%, (c) 4%, (d) 2%, (e) 2%, (f) 2%, (g) 2%, (h) 3%]

For a Newtonian fluid of constant pressure and constant thermal conductivity, k, the equation of thermal energy with a non-negligible viscous heating may be written as:

$$\rho \hat{C}_p \frac{DT}{Dt} = k\nabla^2 T + \mu\Phi_v \ ,$$

where D/Dt is the substantial derivative, ρ the fluid density, $\hat{C}_p$ the heat capacity per unit mass, μ the fluid viscosity, and Φ_v the dissipation function (note: $\mu\,\Phi_v = -\underline{\underline{\tau}} : \underline{\nabla v}$). The term $\underline{\underline{\tau}}$ is the shear stress tensor and $\underline{\nabla v}$ is the velocity gradient tensor. Now, consider a Newtonian fluid flowing in between two large parallel plates as depicted in the following figure. The lower plate is fixed and maintained at a temperature T_0, while the upper plate is moving at a constant velocity V and maintained at a higher temperature T_b. The two plates are separated with a distance of b. Assume that the viscous heating effect is not negligible, and all relevant physical properties can be taken constant.

(a) Give the SI unit of $\mu\,\Phi_v$.

(b) Derive the velocity profile for the fluid and give the resulting expression for $\mu\,\Phi_v$.

(c) Derive the temperature profile for the fluid with the viscous heating taken into account. Express the profile in terms of the dimensionless temperature Θ ($=(T-T_0)/(T_b-T_0)$) and dimensionless distance η ($=y/b$).

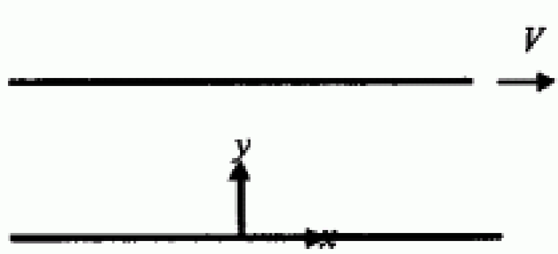
(d) There will arise a dimensionless group Br ($=\mu V^2/(k(T_b-T_0))$) in your answer to part (c). Give the physical meaning of this dimensionless group.

(e) Under what conditions, will there be a maximum in the fluid temperature distribution?

(f) Give the expressions for the normal heat fluxes at the lower and upper plates.

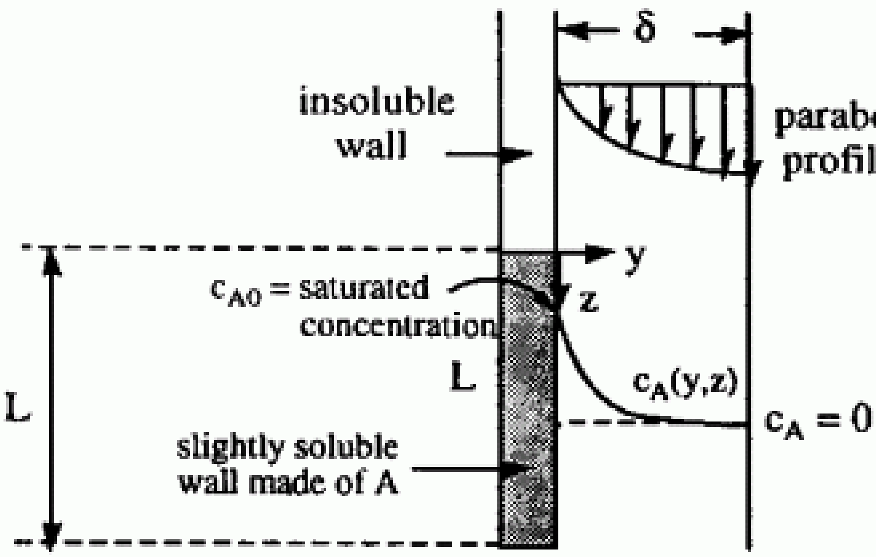
(g) The normal heat flux at the lower plate is found to be always negative. Explain this result physically.

(h) Under a specific condition, the normal heat flux at the upper plate will become positive. What is this condition? Also, explain this result physically.



Problem 3 [20%]

Liquid B of density ρ and viscosity μ is flowing in laminar motion down a vertical wall. For $z < 0$, the wall does not dissolve in the fluid, but for $0 < z < L$, the wall contains a species A that is slightly soluble in B. The film begins far enough up the wall so that v_z depends only on y:



(a) Show that if the "contact time", L/v_{max}, is small (v_{max} = maximum velocity of the fluid), then the equation of continuity of substance A can be approximated by

$$ay\frac{\partial c_A}{\partial z} = D_{AB}\frac{\partial^2 c_A}{\partial y^2}$$

where $a = \rho g \delta / \mu$. Here, g is the acceleration of gravity and D_{AB} is the binary mass diffusivity. State the boundary conditions to be used to solve this equation.

(b) The above differential equation may be solved by letting

$$c_A/c_{A0} = f(\eta)$$

with

$$\eta = y\left(\frac{a}{9D_{AB}z}\right)^{1/3}$$

Show that the differential equation may be expressed in terms of η and f as

$$\frac{d^2f}{d\eta^2} + 3\eta^2\frac{df}{d\eta} = 0$$

Express the boundary conditions in terms of η and f.

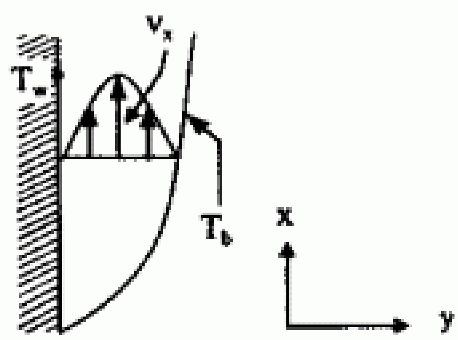
Problem 4 [(a)15%, (b)5%]

Dimensionless groups are used extensively in the correlations for fluid flow and heat transfer.

(a) Please apply the techniques of dimensional analysis to obtain dimensionless groups for the problem of "natural convection heat transfer outside a vertical plane"

Your result will be Nu = f(Gr, Pr) .

(b) Also please discuss the physical meaning of these three dimensionless groups. (e.g., Re = d u ρ / μ = inertia force/viscous force)



For your reference, the following equations are the momentum and energy balance equations for this situation. They should be helpful to start your dimensional analysis.

$$\rho(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}) = g(\rho_b - \rho) + \mu \frac{\partial^2 v_x}{\partial y^2}$$

$$\beta = \frac{\rho_b - \rho}{\rho(T - T_b)}$$

$$\rho C_p(v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y}) = k \frac{\partial^2 T}{\partial y^2}$$

[Notation: ρ = density, v = velocity, μ = viscosity, β = thermal expansion coefficient, k = thermal conductivity, Cp = heat capacity, T = temperature, g = acceleration of gravity, subscript b = bulk, d = characteristic length, u = characteristic velocity, Nu = Nusselt no.,Gr = Grashof no., Pr = Prandtl no., Re = Reynolds no.]

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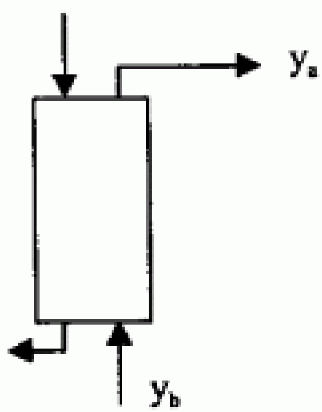
九十一學年度 化學工程學 系（所） 組碩士班研究生招生考試

科目 輸送現象及單元操作 科號 1201 共 5 頁第 5 頁 *請在試卷【答案卷】內作答

Problem 5 [(a)12%, (b)6%, (c)2%]

(a) In a gas absorption operation, the solute concerned is dilute in both phases. Also, Henry's law is assumed to be applicable ($y^*=mx$). Show that (i) the overall mass transfer coefficient K_y can be evaluated by: $1/K_y = 1/k_y + m/k_x$ where k_y and k_x are film mass transfer coefficients of gas and liquid phases; (ii) the overall number of transfer unit based on gas side, No_y , is $(y_b-y_a)/(\Delta y_{LM})$, where (Δy_{LM}) is the log mean value of the terminal Δy's(i.e., $y-y^*$ at the entrance and exit of the column).



(b) A gas stream containing 2% A is absorbed by pure liquid B to remove 99% of A. The gas rate is 20 mol/s, and the equilibrium data can be described by $y^*=3.1x$. Calculate the minimum liquid rate.

(c) Explain how chemical reaction can help gas absorption.