國 立 清 華 大 學 命 題 紙

95 學年度_____化學工程學____系（所）________組碩士班入學考試

科目 輸送現象及單元操作 科目代碼 1401 共 4 頁第 1 頁 *請在【答案卷卡】內作答

1. (a)Derive the equation of motion in tensor form:

$$\frac{\partial}{\partial t}\rho\mathbf{v} = -[\nabla\cdot\rho\mathbf{v}\mathbf{v}] - \nabla p - [\nabla\cdot\boldsymbol{\tau}] + \rho\mathbf{g} \qquad (1) \qquad (5\%)$$

(b)Take the dot product of the velocity vector **v** with the equation of motion and introduce the potential energy (per unit mass) $\hat{\phi}$, defined by

$$\mathbf{g} = -\nabla\hat{\phi}$$

derive the following equation of change for mechanical energy:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \rho\hat{\phi}\right) = -\left(\nabla\cdot\left(\frac{1}{2}\rho v^2 + \rho\hat{\phi}\right)\mathbf{v}\right)$$
$$-(\nabla\cdot p\mathbf{v}) - p(-\nabla\cdot\mathbf{v}) - (\nabla\cdot[\boldsymbol{\tau}\cdot\mathbf{v}]) - (-\boldsymbol{\tau}:\nabla\mathbf{v}) \qquad (2) \qquad (5\%)$$

(c)What is the physical meaning of the terms in Eq.(1) and Eq.(2) (5%)

(d)For constant ρ and μ, insert the Newtonian expression for $\boldsymbol{\tau}$ from Eq.(3)

$$\tau = -\mu(\nabla\mathbf{v} + (\nabla\mathbf{v})^{+}) + \left(\frac{2}{3}\mu - \kappa\right)(\nabla\cdot\mathbf{v})\boldsymbol{\delta} \qquad (3)$$

to develop the following well-known Navier-Stokes equation:

$$\rho\frac{D}{Dt}\mathbf{v} = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g} \qquad (4) \qquad (5\%)$$

2. (a) Put the following four substances in order of increasing thermal conductivity and give the reasons for your answer: aluminum (100 °C), brick (20 °C), aluminum (20 °C), brick (100 °C). (4%)

(b) Which of the following equations most accurately describes the transient decay of the temperature of a fluid-filled tank. The tank (mass m, surface area A, overall heat transfer coefficient U, specific heat capacity including contents, $\hat{C}_p$) is initially at T_0, and the ambient air temperature is T_∞.

(i) $\dfrac{T(t)-T_\infty}{T_0-T_\infty}=\exp(-\dfrac{m\hat{C}_p}{UA}t)$

(ii) $\dfrac{T(t)-T_\infty}{T_0-T_\infty}=\exp(-\dfrac{UA}{m\hat{C}_p}t)$

(iii) $\dfrac{T(t)-T_0}{T_0-T_\infty}=\exp(-\dfrac{m\hat{C}_p}{UA}t)$

(iv) $\dfrac{T(t)-T_0}{T_0-T_\infty}=\exp(-\dfrac{UA}{m\hat{C}_p}t)$

Give the reasons for your choice. (6%)

(c) A triangular fin (equilateral cross-section, side length b, length L; $b \ll L$) extends from a solid wall into a fluid. The wall is at temperature T_w, and the fluid is at temperature T_a. Heat is transferred from the fin to the fluid according to the flux expression

$q_{\text{fin-fluid}} = h(T - T_a)$.

Here, h is the heat transfer coefficient. Assume that the temperature in the fin depends only on z (the direction normal to the solid wall) and all relevant physical properties, such as the fin density ρ, specific heat capacity $\hat{C}_p$, and thermal conductivity k and h, are independent of temperature.

(i) Use a shell energy balance to obtain a differential equation for the unsteady-state temperature distribution in the fin, $T(z,t)$ in the following form

$$A\frac{dT}{dt}=\frac{d^2T}{dz^2}+B(T-T_a)$$

Give the expressions for A and B. Note that the area of an equilateral triangle of side length b is $\dfrac{\sqrt{3}}{4}b^2$. (6%)

(ii) At steady state, give the appropriate boundary conditions for $T(z)$. (4%)

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3.(a)For abinary diffusion, show that the corresponding Fick's first law can also be expressed as

$$c(\mathbf{v}_A - \mathbf{v}_B) = -\frac{cD_{AB}}{x_A x_B}\nabla x_A$$

where c is the total molar concentration, $\mathbf{v}_i$ is the velocity of species i (i = A or B), D_{AB} is the diffusion coefficient, x_i is the mole fraction of species i. (5%)

(b)Starting with the equation of continuity of component A in a binary mixture of A and B:

$$\frac{\partial \rho_A}{\partial t} + \nabla \cdot \mathbf{n}_A = r_A$$

where ρ_A is the mass concentration of A, $\mathbf{n}_A$ is the combined mass flux of A and r_A is the production of mass of A per unit time per unit volume by chemical reactions, convert this to the following for constant total mass concentration ρ:

$$\frac{\partial \rho_A}{\partial t} + \mathbf{v} \cdot \nabla \rho_A - \nabla \cdot D_{AB} \nabla \rho_A = r_A$$

where **v** is the mass average velocity of the mixture. (5%)

(c)An early mass-transfer study of oxygen transport in human tissue won a Nobel prize for August Krough. By considering a tissue cylinder surrounding each blood vessel, he proposed that the diffusion of oxygen away from the blood vessel into the annular tissue (with the inner radius of R_1 and the outer radius of R_2) was accompanied by a zero-order reaction; that is, R_{O2} =-*m*, where *m* is a constant. The reaction was necessary to explain the metabolic consumption of the oxygen to produce carbon dioxide. Since the tissue cylinders were believed to be arranged in a hexagonal bundle, the following boundary conditions were suggested:

at $r=R_1$, $P_{O2}=P_{O2,1}$, the average oxygen pressure value between the arterial and venous ends of the blood vessel

at $r=R_2$, $dP_{O2}/dr=0$, due to the identical oxygen fluxes from the neighboring tissue cylinders

Determine the concentration (in terms of P_{O2}) profile as a function of r, and the flux of oxygen that enters the tissue cylinder. (10%)

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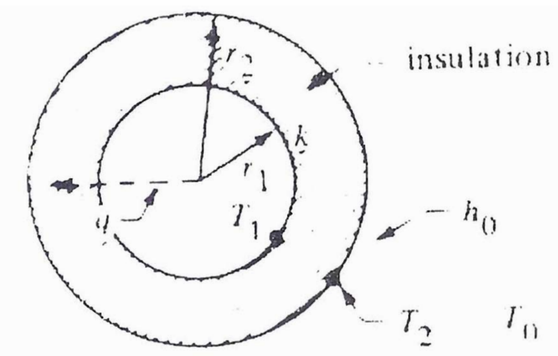
4. (a) Consider a layer of insulation around the outside of a cylinder of radius r_1 and length L. The cylinder has a temperature T_1 at r_1 (as shown in the following figure). The outer surface of insulation is at T_2 and is exposed to an environment of T_0 where convective heat transfer occurs. When insulation is added, it decreases T_2 but increases surface area for heat loss. As a result, there is a critical thickness where heat loss is at a maximum. Please derive the equation for calculation of this critical thickness. (8%)

(b) An electric wire of diameter 1.5 mm, covered with a plastic insulation (thickness = 2.5 mm) is exposed to air at 300K and ho = 20 W/m^2-K. The insulation has a k of 0.4 W/m-K. It is assumed that the surface temperature of wire is constant at 400K. Calculate the heat loss per meter of wire, heat loss after insulation and the critical thickness of insulation for this case. (12%)



5.(a)Briefly discuss the following questions:

(i)Which type of cake is easier for filtration operation, compressible or incompressible? Explain. (2%)

(ii)What is critical moisture? How is it used in the derivation of drying rate? (3%)

(iii)What is breakthrough curve in adsorption operation? Discuss the relationship between its shape and the performance of a column. (5%)

(b)An antibiotic is to be extracted from water to solvent at pH 2.5, where K_D (partition coefficient of the antibiotic between solvent and water) = 15. If the solvent flow rate is set at 0.5 times the water rate, how many ideal stages would be required for 95% recovery? (10%)