

八十五學年度 生命科學 系(所) 分生組乙 生醫組乙 組碩士班研究生入學考試
 科目 微積分 科號 1005 1305 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1. (10%) Let f be a function defined at $t = t_0 \in [a, b]$
 - (a) (6%) Prove the following statement;
 f is differentiable at $t_0 \implies f$ is continuous at t_0
 $\implies f$ has limit at t_0
 - (b) (4%) Disprove by giving counterexamples that the implication " \implies " are false.

2. Prove the followings;
 - (a) (10%) The perimeter of a circle of radius r is $2\pi r$.
 - (b) (5%) The area of a circle of radius r is πr^2 .
 - (c) (10%) The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.
 - (d) (10%) the surface area of a sphere of radius r is $4\pi r^2$.

3. (15%) Evaluate $\int \frac{-2x+4}{(x^2+1)(x-1)^2} dx$ by partial fractions.

4. (10%) show that the equation $x^4 + 3x + 1 = 0$ has exactly one solution in $[-2, -1]$.

5. (15%)
 - (a) Show that

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots$$
 - (b) Find the radius of convergence of the above power series.
 - (c) Evaluate π to two decimal from (a).

6. (15%) State without proof of Green's theorem, Divergence theorem, Stokes's theorem.