

八十六學年度 生命科學 系(所) 丙 組碩士班研究生入學考試
 科目 應用數學 科號 1105 共 / 頁第 / 頁 *請在試卷【答案卷】內作答

1. Find the first two terms of the asymptotic series for $\int_1^{\infty} e^{-t} dt$. (20%)

2. Find the following derivatives: (20%)

(a) $(d^{25} / dx^{25})(x \cos x)$

(b) $(d^{100} / dx^{100})(x^2 e^{-x})$

3. Let $h(x) = \sum_{k=-\infty}^{\infty} f(x + 2k\pi)$, assuming that the series converges to a function satisfying Dirichlet conditions and has period 2π , then (20%)

(a) expand $h(x)$ in an exponential Fourier series $h(x) = \sum_{n=-\infty}^{\infty} c_n \exp(inx)$;

show that $c_n = F(n)$ where $F(\alpha)$ is a Fourier transform of $f(x)$.

(b) show that $\sum_{k=-\infty}^{\infty} f(2k\pi) = \sum_{n=-\infty}^{\infty} F(n)$

4. Evaluate the following integrals: (20%)

(a) $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx$

(b) $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

5. let $z = x + iy$, show that if $f(z) = u(x, y) + iv(x, y)$ is analytic in a region, then: (20%)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$