

國立清華大學命題紙

八十四學年度 工業工程所甲.乙.組碩士班研究生入學考試

科目 微積分與線性代數 科號 ²⁷⁰²~~2802~~ 共 2 頁第 / 頁 *請在試卷【答案卷】內作答

1. (a) Differentiate $y = (\sin x^2)^{\exp x^2}$ (6%)

(b) Differentiate $y = \sqrt{\frac{1-x}{1+x}}$ (6%)

(c) Differentiate $y = \sqrt{\frac{a^2+x^2}{a^2-x^2}} \sqrt[3]{\frac{a^2-x^2}{a^2+x^2}}$ (6%)

2. (a) Calculate $\int \frac{\ln \ln x^2}{x \ln x^2} dx$ (6%)

(b) Calculate $\int \sqrt{1-x^2} dx$ (6%)

3. (a) Determine $\lim_{x \rightarrow 1^-} \frac{[x^2]-1}{x^2-1}$ where $[x]$ is the integer such that $[x] \leq x < [x]+1$ (5%)

(b) Find the first and second differential coefficients of $y = \frac{x}{b} \sqrt{(a-x)x}$ (5%)

4. Let $S_n = 1 - \frac{1}{3} + \frac{1}{8} - \frac{1}{15} + \dots + (-1)^n \frac{1}{n(n+2)}$.

(a) Determine $S = \lim_{n \rightarrow \infty} S_n$ and (b) find out the first n such that $|S_n - S| < \frac{1}{1000}$.

(10%)

5. Let A be an invertible matrix. Show that A^{-1} is unique. (10%)

6. $A = \begin{bmatrix} .5 & .2 & .3 \\ .3 & .8 & .3 \\ .2 & .0 & .4 \end{bmatrix}$, $u_1 = \begin{bmatrix} .3 \\ .6 \\ .1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $v_0 = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$. u_1 and u_2 are eigenvectors of A .

(a) Find another eigenvector of A : u_3 , so that u_1 , u_2 , and u_3 form a basis for \mathbb{R}^3 . (5%)

(b) Write v_0 as a linear combination of u_1 , u_2 , and u_3 . (5%)

(c) For $k = 1, 2, 3, \dots$, define a dynamic system: $v_k = A^k v_0$. Find the (vector) value that v_k will converge to when k increases. (5%)

八十四學年度 工業工程 系 甲、乙 組碩士班研究生入學考試

科目 微積分與線性代數 共 2 頁第 2 頁 *請在試卷【答案卷】內作答

7. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation such that

$$T(x_1, x_2) = (x_1 + 2x_2, -x_1 - 3x_2, -3x_1 - 2x_2).$$

Find \mathbf{x} such that $T(\mathbf{x}) = (-4, 7, 0)$. (10%)

8. The matrix below are row equivalent.

$$A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- (a) Find rank A and $\dim \text{Nul } A$. (5%)
- (b) Find bases for $\text{Col } A$ and $\text{Row } A$. (5%)
- (c) Find a basis for $\text{Nul } A$. (5%)