

八十五學年度 工業工程 系(所) 工甲、乙 組碩士班研究生入學考試
 科目 微積分與線性代數 科號 3202 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. (a) Try to find an orthonormal basis for the subspace V_0 of \mathbb{R}^4 spanned by

$$v_1 = [1 \ 1 \ 1 \ -1]^T, \quad v_2 = [2 \ -1 \ -1 \ 1]^T$$

$$v_3 = [0 \ 3 \ 3 \ -3]^T, \quad v_4 = [-1 \ 2 \ 2 \ 1]^T \quad (10\%)$$

(b) Find an orthonormal projection P onto V_0 . (10%)

2. (a) Let $A = QR$ be a normalized QR-decomposition of the $p \times q$ matrix A . Show that all solutions to the least-square problem of finding x to $\min \|Ax - y\|_2$ can be obtained by applying back-substitution to solve $Rx = Q^T y$. (10%)

(b) Using the result of (a) to solve the least-square problem:

$$Ax \approx y = [1 \ -1 \ 2 \ 1]^T$$

when $A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 1 & -1 & 3 & 2 \\ 1 & -1 & 3 & 2 \\ -1 & 1 & -3 & 1 \end{pmatrix}$

(10%)

3. Discuss the solution set of the following system of equations with α an arbitrary parameter.

$$x - 3y = -2$$

$$3x - 2y = \alpha$$

$$2x + y = 3$$

(10%)

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科目 微積分與線性代數 科號 $\frac{320}{330}$ 共 3 頁第 2 頁 *請在試卷【答案卷】內作答

4. 單一選擇題，每題四分，請依題號順序作答，將答案寫在“答案卷”上。

(20%)

(4-1) The maximum point of the graph of the equation $f(x) = x + \sin x$ is

- (a) 0 (b) $\pi/6$ (c) $\pi/3$ (d) $\pi/2$ (e) π
 (f) none of the above.

(4-2) Let $F(x) = (e^{3x} - 5x)^{(1/x)}$, $\lim_{x \rightarrow 0} F(x) =$

- (a) e^{-1} (b) e^{-2} (c) e^1 (d) e^2 (e) e^3
 (f) none of the above.

(4-3) $[y]$ is defined to be the greatest integer $\leq y$. $\int_0^{10} [x/3] dx =$

- (a) 15 (b) 12 (c) 10 (d) 8 (e) 1
 (f) none of the above.

(4-4) The domain of convergence of the series: $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2^n (3n-1)}$ is

- (a) $-1 \leq x \leq 3$ (b) $-1 \leq x \leq 2$ (c) $-1 \leq x \leq 1$ (d) all $x \neq 0$
 (e) $x > 0$ (f) none of the above.

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科目 微積分與線性代數 科號 $\frac{3202}{3302}$ 共 3 頁第 3 頁 *請在試卷【答案卷】內作答

$$(4-5) \lim_{x \rightarrow \pi/2} \frac{ex^2/\pi - e\pi/4 + \int_x^{\pi/2} e^{\sin t} dt}{1 + \cos 2x} =$$

- (a) $\pi/2$ (b) π/e (c) e/π (d) $2e/\pi$ (e) $e/2\pi$
 (f) none of the above.

5. Evaluate $\int_0^x \frac{x \sin x}{1 + \cos^2 x} dx$. (10%)

6. (6-1) Sketch the 3 dimensional region R bounded by $x + y + z = a$ ($a > 0$),
 $x = 0$, $y = 0$, $z = 0$
 (6-2) Evaluate the triple integral

$$\iiint_R (x^2 + y^2 + z^2) dx dy dz. \quad (10\%)$$

7. Let $U = x^3 y$. Find dU/dt if (i) $x^5 + y = t$, and (ii) $x^2 + y^3 = t^2$. (10%)