國 立 清 華 大 學 命 題 紙

八十五學年度 工業工程 系（所）工工 甲、乙、丙、工管一般 組碩士班研究生入學考試

科目 機率與統計 科號 3201, 3301 3402, 3601 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. (21%) Let X be the number of defectives in a sample of size n. Then X is a binomial random variable with parameter n (number of trials) and p (defective rate). Suppose the defective rate p follows beta distribution with parameters α and β (Recall the beta probability density function (pdf) is proportional to $p^{\alpha-1}(1-p)^{\beta-1}$.)

   (a) Write $f_{X|P}(x|p)$, which is the conditional pdf of X given p.

   (b) Write $f_P(p)$, which is the pdf of P.

   (c) Compute $f_{X,P}(x,p)$, the joint pdf of X and P.

   (d) Compute $f_X(x)$, the marginal pdf of X.

   (e) Compute $f_{P|X}(p|x)$, the conditional pdf of P, given the value of x.

   (f) Compute $E(p|x)$.

   (g) Give the Bayes estimate of p.

2. (10%). The golf scores of two competitors, A and B, are recorded over a period of 10 days. Golfer A claims that her game is better than that of B. Use the following data to test this claim. Assume that the playing conditions on different days are different. Make any additional assumptions if necessary.

days	Golfer A	Golfer B
1	43	51
2	82	84
3	77	74
4	79	82
5	39	48
6	51	53
7	66	61
8	55	59
9	61	75
10	43	48

3. (12%)

   (a) Explain the law of large number and the Central Limit Theorem.

   (b) Explain how the Central Limit Theorem can be applied in a real situation.

4. (10%) Let X and Y be independent and identically distributed random variables, where the cumulative density function (cdf) of X and Y are listed as follows:
$F_X(t) = F_Y(t) = 1 - e^{-\lambda t}$ for all $t \geq 0$.
Find the cdf of $X + Y$.

5. (15%) An engineer approximates the reliability of a cutting assembly by

$$R(t) = \begin{cases} (1 - t/t_0)^2 & 0 \leq t \leq t_0 \\ 0 & t \geq t_0 \end{cases}$$

(a) Determine the failure rate (Hint. $\lambda(t) = \frac{f(t)}{R(t)}$).

(b) Does the failure rate increases or decrease with time?

(c) Determine MTTF (mean time to failure).

6.(12%) 某公司被檢舉對男女從業員在薪資上有明顯的差別待遇，經抽訪數位男女作業員後，得其起薪之統計資料如下：

男	女
$n_1 = 11$	$n_2 = 14$
$\overline{X_1} = 20{,}400$	$\overline{X_2} = 19{,}850$
$\hat{s}_1 = 1{,}100$	$\hat{s}_2 = 1{,}350$

假設母體皆為常態分配
(a) 試以 $\alpha = 0.1$ 檢定二母體之變異數是否相等？
(b) 試以 $\alpha = 0.1$ 檢定男女作業員薪資是否有明顯差異？

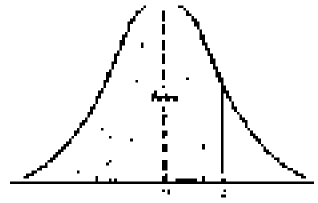
7.(10%) 假設X, Y之聯合機率分配為

$$f(x,y) = \begin{cases} e^{-2x}, & 0 < y < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

試求Y對X之迴歸方程式為何？

8.(10%) 在某次考試中，甲生的成績為統計66分，微積分82分，而統計在全班的平均分數為50分，標準差為11分，微積分在全班的平均為73分，標準差為9分，假設二個考試中考生皆為相同一批人，請問甲生在那一科中的表現較為傑出？

率与统计　共3页 第3页



3 Areas Under the Normal Curve

.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

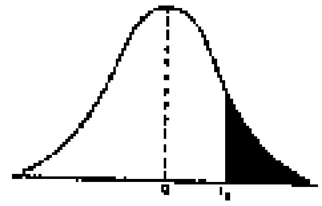


Table A.4* Critical Values of the t-Distribution

v	α = 0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
inf.	1.282	1.645	1.960	2.326	2.576

* From Table IV of R. A. Fisher, *Statistical Methods for Research Workers*, published by Oliver & Boyd, Edinburgh, by permission of the author and publishers.

Table A.6* Critical Values of the F-Distribution

$f_{0.05}(v_1, v_2)$

v_2	v_1 = 1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

Table A.6 (*continued*) Critical Values of the F-Distribution

$f_{0.05}(v_1, v_2)$

v_2	v_1 = 10	12	15	20	24	30	40	60	120	∞
1	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00