

八十五學年度 數學系(所)應用數學組碩士班研究生入學考試
 科目 數值分析 科號 0204 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (16%)

(a) Derive the approximation formula

$$f'(x) \approx \frac{1}{2h} [4f(x+h) - 3f(x) - f(x+2h)]$$

and show that its error term is of the form $\frac{1}{9}h^2 f'''(\xi)$.

(b) Base on above formula, use Richardson extrapolation to obtain an approximation formula of $f'(x)$ which error term is of $O(h^3)$.

2. (16%) Let $A \in \mathbb{R}^{n \times n}$. If we do one step of Gaussian elimination on A , then we obtain a matrix $A_1 \in \mathbb{R}^{n \times n}$. Show that A_1 can be expressed in the following form

$$A_1 = MA$$

where $M = I_n - me_1^T$,
 I_n is the identity matrix,
 $m = (0, m_2, \dots, m_n)^T \in \mathbb{R}^n$
 $e_1 = (1, 0, \dots, 0)^T \in \mathbb{R}^n$

3. (16%) Let $p_k(x)$ be the polynomial of degree $\leq k$ interpolating $f(x)$ at x_0, x_1, \dots, x_k and $q(x)$ be the polynomial of degree $< k-1$ interpolating $f(x)$ at x_1, x_2, \dots, x_k .

(a) Show that

$$p_k(x) = q(x) + \frac{x - x_k}{x_k - x_0} [q(x) - p_{k-1}(x)]$$

(b) Form (a) prove the formula of divided difference,

$$f[x_0, x_1, \dots, x_k] = \frac{f[x_1, x_2, \dots, x_k] - f[x_0, x_1, \dots, x_{k-1}]}{x_k - x_0}$$

4. (16%) Define a sequence $x_{n+1} = x_n - \tan x_n$ with $x_0 = 3$. What is $\lim_{n \rightarrow \infty} x_n$? Show that the sequence $\{x_n\}$ converges quadratically.

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5. (16%)

(a) Derive Simpson's rule

$$\int_a^{a+2h} f(x) dx \approx \frac{h}{3} [f(a) + 4f(a+h) + f(a+2h)].$$

(b) For the initial valued problem $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$, from (a) derive a two-step implicit numerical scheme and explain how to apply the scheme in practice.