九十學年度 數學 系（所） 純數 組碩士班研究生招生考試

高等微積分 科號 0101 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1. (15 points) Find the maximum and minimum of the function $f(x, y, z) = x + 3y + 5z$ on the region $\{(x, y, z) : x^2 + y^2 \le z \le 5\}$.

2. (15 points) Let f be a one to one continuous function on $[0, 1]$. Show that f is either strictly increasing or strictly decreasing.

3. (15 points) If $C \subset \mathbf{R}^n$ is connected, show that its closure $cl(C)$ is also connected.

4. (15 points) Let $f : [0, 1] \to \mathbf{R}$ be a continuous function, consider the sequence of functions

$$f_0(x) = f(x),\ f_{n+1}(x) = \int_0^x f_n(t)dt,\ n = 0, 1, 2, 3, \cdots,\ x \in [0, 1].$$

Show that $\sum_{n=0}^{\infty} f_n(x)$ converges uniformly on $[0, 1]$.

5. (15 points) Let $f : [a, b] \to \mathbf{R}$ be continuous, and

$$F(x) \equiv \int_a^b f(y)|x - y|dy.$$

Find $F''(x)$.

6. (15 points) For what values of $r \in \mathbf{R}$ is

$$\int_0^{\infty} x^r e^{-x} dx$$

convergent? Verify your answer.

7. (15 points) A real value function $f(x)$ on (a, b) is a convex function if

$$f(\lambda c + (1 - \lambda)d) \le \lambda f(c) + (1 - \lambda)f(d)$$

for all $a < c < d < b$ and $0 \le \lambda \le 1$. Prove that f is a differentiable convex function on (a, b) iff $f'(x)$ is increasing on (a, b).

8. (15points) Let $S = \{(x, y, z) : x^2 + y^2 + z^2 = 1,\ z \ge 0\}$ and $\overrightarrow{F} = y^2 z^3 \overrightarrow{i} + x^4 z \overrightarrow{j} + (x^2 + y^2) \overrightarrow{k}$. Evaluate the surface integral

$$\iint_S \overrightarrow{F} \bullet \overrightarrow{n} dS$$

where $\overrightarrow{n}$ is the unit normal vector of S pointing outward.