九十學年度 科技管理研究所 系（所） 乙 組碩士班研究生招生考試

微積分 科號 5603 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1. (10 points) Find the limits. (5 points for each problem)

(i) $\lim_{n\to\infty} n(a^{\frac{1}{n}} - 1)$, where $a > 0$ (ii) $\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{k^2}{n^3}\right)$

2. (10 points) Evaluate the following integrals. (5 points for each problem)

(i) $\int_0^1\int_x^1 e^{y^2}\,dydx$ (ii) $\int\frac{x^2}{(x-1)^2(x+1)}\,dx$

3. (10 points) Determine the convergence or divergence of the following series. Show your reasons. No credit will be granted if you just answer "convergence" or "divergence". (5 points for each problem)

(i) $\sum_{k=10}^{\infty}\frac{1}{\ln(\ln k)}$ (ii) $\sum_{k=1}^{\infty}\frac{k!}{k^k}$

4. (10 points) Calculate the derivative

$$\frac{d}{dx}\left(\int_{\sqrt{x}}^{x^2+x}\frac{dt}{2+\sqrt{t}}\right).$$

5. (10 points) Let $C: x^3 + y^3 - 2xy = 0$ be a curve on the xy-plane through the point (1,1). Find the equation of the tangent line L to C at (1,1).

6. (10 points) Calculate the area enclosed by the curve C defined by the polar equation $r = a(1+\cos 3\theta)$ from $\theta = -\frac{\pi}{3}$ to $\theta = \frac{\pi}{3}$, where $a > 0$.

7. (10 points) Find the length of the arc C: $\overrightarrow{r}(t) = t\hat{\mathrm{i}} + \frac{2}{3}\sqrt{2}t^{\frac{3}{2}}\hat{\mathrm{j}} + \frac{1}{2}t^2\hat{\mathrm{k}}$ from $t = 0$ to $t = 2$, where $\hat{\mathrm{i}} = (1,0,0)$, $\hat{\mathrm{j}} = (0,1,0)$ and $\hat{\mathrm{k}} = (0,0,1)$ in $\mathbf{R}^3$.

8. (10 points) Find the surface area of that part of the parabolic cylinder $z = y^2$ that lies over the triangle with vertices (0,0), (0,1) and (1,1) in the xy-plane.

9. (10 points) Minimize the function $f(x,y) = 3x + y + 10$ subject to the constraint $x^2y = 6$.

10. (10 points) Evaluate the following surface integral

$$\iint_{S^2}(x^4+y^4+z^4)\,d\sigma,$$

where $S^2 = \{(x,y,z)\in\mathbf{R}^3 | x^2+y^2+z^2 = 1\}$ is the unit sphere in $\mathbf{R}^3$ and $d\sigma$ is the surface element on S^2.