

一 填充題(共9題,第1至第4每題7分,第5至第9每題8分共68分)

1. $\lim_{n \rightarrow \infty} n \ln \left(\frac{\sqrt[3]{2} + \sqrt[3]{3}}{2} \right) = \underline{\hspace{2cm}}$.

2. $\lim_{n \rightarrow \infty} \frac{1 + \sqrt{2} + \sqrt{3} + \dots + \sqrt{n}}{n\sqrt{n}} = \underline{\hspace{2cm}}$.

3. $\int_0^1 \frac{dx}{1 + \sqrt[3]{x}} = \underline{\hspace{2cm}}$.

4. If $g(x, y) = \begin{cases} x & \text{for } x \geq y \\ y & \text{for } x < y, \end{cases}$ then $\int_0^\pi g(\cos t, \sin t) dt = \underline{\hspace{2cm}}$.

5. Suppose x and y are differentiable functions of t and $\frac{dy}{dx} = \sin 2t$,
 $\frac{d^2y}{dx^2} = 2(\sin t + \cos t)$. Then $\frac{dx}{dt} = \underline{\hspace{2cm}}$.

6. A circle of radius 1 with center on the y -axis is inscribed in (內切於) the parabola $y = x^2$. The points of intersection are $\underline{\hspace{2cm}}$.

7. Let $\sum_{n=0}^{\infty} a_n (x - \pi)^n$ be the power series expansion of $x \sin x$. Then
 $a_3 + a_4 = \underline{\hspace{2cm}}$.

8. $\int_0^{\sqrt{\pi}/2} \int_y^{\sqrt{\pi}/2} \cos(x^2) dx dy = \underline{\hspace{2cm}}$.

9. Suppose $f : [0, \infty) \rightarrow \mathbf{R}$ is a continuous function such that $f(0) = 1$
and $f(x) = \frac{1}{x} \int_0^x (t^2 + 1) f(t) dt$ for all $x > 0$. Then $f(x) = \underline{\hspace{2cm}}$.

二 計算與證明(三題,共32分;必須寫出計算證明過程)

(8%) 1. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of positive real numbers. Suppose
 $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = r < 1$. Prove that $\sum_{n=1}^{\infty} a_n$ converges.

(12%) 2. Find the maximum and minimum values of the function $f(x, y) =$
 $2x^2 + y^2 - xy$ on the unit disk $x^2 + y^2 \leq 1$.

- (12%) 3. 某遊覽公司提供一部50個座位的巴士到日月潭旅遊。遊客至少16人才發車。若遊客總數在16人至30人之間，則每人車資170元。若超過30人，則每超出一人所有的遊客每人車資減少3元。試問此巴士載多少遊客時，遊覽公司有最多收入？