

八十四學年度 物理研究所 物理 組碩士班研究生入學考試
 科目 應用數學 科號 0403 共 二 頁第 一 頁 *請在試卷【答案卷】內作答
0503

1. Find the orthogonal trajectories of

$$y = \frac{cx}{1+x}$$

where c is a constant. (10%)

2. Find the general solution of the following differential equations:

(a)

$$x^2 y'' + xy' - y = 4 \quad (7\%)$$

(b)

$$y'' + y = 4x + 10 \sin x \quad (8\%)$$

3. Calculate the Laplace transform of $2t + 3$. (5%)

4. Calculate the Fourier transform of

$$f(x) = \exp(x) \quad \text{if } x < 0$$

$$= 0 \quad \text{if } x > 0. \quad (5\%)$$

5. Find the Fourier series of the function which is assumed to have period 2π

$$f(x) = \pi - x \quad (0 < x < 2\pi). \quad (10\%)$$

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6. Let a, b, c be the eigenvalues of the matrix

$$\begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

Calculate $a + b + c$ and $a^2 + b^2 + c^2$. (10%)

7. Evaluate the integrals

(a)

$$\int_{-\infty}^{\infty} \frac{x^2 - 1}{x^4 + 5x^2 + 4} dx \quad (10\%)$$

(b)

$$\int_0^{2\pi} \frac{\sin \theta}{3 + \cos \theta} d\theta. \quad (10\%)$$

8. The parametric representations of two straight lines are

$$\vec{r}_1 = (t, t, 2t)$$

$$\vec{r}_2 = (1 - s, 2 + s, 1 + 2s)$$

where t and s are parameters, find the shortest distance between these two lines. (10%)

9. Find the linear fractional transformation that maps the points $z_1 = -2, z_2 = 0, z_3 = 2$ into the points $w_1 = \infty, w_2 = 2, w_3 = 3$ respectively. (5%)

10. Calculate the volume of a 4-dimensional unit sphere. (10%)