

八十五學年度 物理 系(所)物理, 應物 組碩士班研究生入學考試

科目 應用數學 科號 <sup>0403</sup>0503 共 2 頁第 1 頁 \*請在試卷【答案卷】內作答

1. Solve the following differential equations:

(a)  $(3x^2 - y^2)y' - 2xy = 0$

(b)  $x^2y'' + 3xy' + y = 0$        $y(1) = 3, \quad y'(1) = -4$

(c)  $y'' - 2y' + 2y = 2e^x \cos x.$       (15%)

2. Find the Laplace transform of the following functions. That is to say: given  $f(t)$ , find  $\mathcal{L}\{f(t)\} = F(s)$ .

(a) the unit step function  $u(t-a) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t > a \end{cases}$

(b) Dirac's delta function  $\delta(t-a) = \begin{cases} \infty & \text{if } t = a \\ 0 & \text{otherwise} \end{cases}$

and  $\int_0^{\infty} \delta(t-a) dt = 1.$       (12%)

3. Determine whether the following linear systems have solutions. If they have, are the solutions unique or infinitely many in number. Also, find the solutions that exist.

(a)  $\begin{cases} 4x + 7y - 2z = 0 \\ -8x + y + 4z = 0 \\ 6x - 9y - 3z = 0 \end{cases}$

(b)  $\begin{cases} x + y + z = 2 \\ 2x - y + 2z = 4 \\ -x + 4y + z = 3 \end{cases}$       (12%)

4. Are the following matrices diagonalizable? For those which are, find a matrix for each, and diagonalize them. For those which are not, explain why.

(a)  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

(b)  $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{pmatrix}$       (12%)

5. (a) What is the condition for a function to be harmonic? Which of the following functions are harmonic?

(i)  $f = x^2 + y^2 + z^2$       (ii)  $f = (x + y)^2$

(b) Show that the integral of the normal derivative of a harmonic function over any piecewise smooth closed orientable surface in the domain of definition is zero.      (12%)

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6. A thin bar, of length 10 cm, is perfectly insulated laterally. Assuming  $c^2 = 4$ , find the temperature distribution  $u(x, t)$  in the bar if both ends are kept at  $0^\circ\text{C}$  and the initial temperature is

$$f(x) = \begin{cases} x & \text{if } 0 < x < 5 \\ 10 - x & \text{if } 5 < x < 10. \end{cases} \quad (12\%)$$

7. Determine where (in the complex plane) the following functions are analytic.

(a)  $f(z) = \frac{x^2}{3} + i(y - \frac{y^3}{3})$

(b)  $f(z) = (1 + i)(x + y^2)$

(c)  $f(z) = z \bar{z}$ . (10%)

8. Evaluate the following integrals:

(a)  $\oint_C e^{1/z} (z - i) dz$  C: unit circle, counterclockwise

(b)  $\int_{-\infty}^{\infty} \frac{x \sin 2x}{x^4 + 4} dx$

(c)  $\oint_C \mathbf{F} \cdot d\mathbf{r}$   $\mathbf{F} = (e^y/x) \mathbf{i} + (e^y \ln x + 2x) \mathbf{j}$   
 C: the boundary of the region  
 $1 + x^4 \leq y \leq 2$ , counterclockwise. (15%)