國立清華大學命題紙

1. (10%) (a) The smallest separation resolvable by a microscope is of the order of magnitude of the wavelength used. What energy electrons would one need in an electron microscope to resolve separations of 100 Å, 10 Å, and 1 Å? (5%) (b) A light emitting diode is constructed from a p-n junction based a certain Ga-As-P semiconducting material, whose energy gap is 1.9 eV. What is the wavelength of the emitting light? (5%)

2. (15%) Consider N free electrons confined to a three-dimensional cube-shaped square well,

$$V(x) = \begin{cases} 0, \text{ for } 0 < x, y, z < L \\ \infty, \text{ elsewhere.} \end{cases}$$

(a) What are the energies and corresponding wave functions of electrons in the box? (4%) (b)Find $D(E)$ (density of states), which is defined as the number of available electronic quantum states per unit energy interval in the vicinity of the energy E, and per unit volume of the well. Keep in mind that there are two electron spin states per energy state. (5%) (c) Express Fermi energy as a function of the electron number density $n = N/L^3$. Given that the number density of free electrons (n) in gold is 6.0×10^{22} cm^{-3}, calculate the Fermi energy in electron volt and the velocity of an electron moving with kinetic energy equal to the Fermi energy. (6%)

3. (10%) In the Bohr Model, the orbital angular momentum (L) of an atomic electron is quantized and $L = m_e vr = n\hbar$ ($n = 1, 2, 3, \cdots$). Prove that the quantization of total energy and orbital of the electron in a hydrogen atom can be obtained by the quantization condition of L. Express your results in terms of $\hbar$, m_e, and a_0 (Bohr radius). (4%) (b) Consider the correction for the finite nuclear mass. Estimate numerically the ground state energy and radius of a hydrogen atom, a positronium atom (a positron and electron pair revolving about their common center of mass), and a deuterium atom [Z=2, M(mass of proton or neutron)$\cong 1836 m_e$]. (6%)

4. (15%) Consider an electron beam of kinetic energy E, incoming from $x = -\infty$, incidents on a rectangular barrier of height V_0 ($V_0 > E$) and width L. The One-dimensional potential barrier can be written as,

$$V(x) = \begin{cases} 0, \; x < 0 \\ V_0, \; 0 \le x \le L \\ 0, \; x > L. \end{cases}$$

(a) What are the electron wave functions in all three regions? What are the boundary conditions at $x = 0$ and $x = L$? (6%)

(b) What is the tunneling coefficient T (probability that the incoming electrons transmit to the right-hand-side of the potential barrier)? What is reflection coefficient R? [Express the results in terms of $k \equiv \sqrt{\frac{2mE}{\hbar^2}}$ and $\alpha \equiv \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$.] (6%)

(c) What is the approximate fraction of incoming electrons that will succeed in penetrating the barrier if $E = 2$ eV, $V_0 = 10$ eV, and $L = 4$ Å? (3%)

5. (15%) (a) Write the Schrödinger equation for a particle of mass m moving in two dimensions under the influence of the anisotropic potential $V(x,y) = \frac{1}{2}m\varpi_x^2x^2 + \frac{1}{2}m\varpi_y^2y^2$, where $\varpi_x \neq \varpi_y$ in general. Use separation of variables method to find the energy levels and corresponding wave functions of this system (in terms of eigenenergies and eigenfunctions of the 1-D simple harmonic oscillator, see appendix). Are these energy levels degenerate?, In the special case of $\varpi_x = \varpi_y = \varpi$, prove that the n^{th} energy level is $(n+1)$-fold degenerate, where $n = 0, 1, 2, 3, \cdots$. (6%) (b) Prove that $\widehat{L}_z = \widehat{x}\widehat{p}_y - \widehat{y}\widehat{p}_x = (\hbar/i)(\partial/\partial\phi)$ in the polar coordinate system (r,ϕ). (3%) (c) Rewrite $V(x,y)$ in terms of polar coordinates r and ϕ and calculate $[\widehat{H}, \widehat{L}_z]$ $(\equiv \widehat{H}\cdot\widehat{L}_z - \widehat{L}_z\cdot\widehat{H})$ commutator. If they don't commute with each other, what is the physical meaning? Explain why? (6%)

6. (15%) (a) Prove that two observables $\widehat{A}$ and $\widehat{B}$ have simultaneous eigenvalues if they commute ($[\widehat{A},\widehat{B}] \equiv \widehat{A}\widehat{B} - \widehat{B}\widehat{A} = 0$). What is the physical consequence of $[\widehat{x}_\mu, \widehat{p}_\nu] = i\hbar\delta_{\mu\nu}$, where $\widehat{x}_\mu$ and $\widehat{p}_\nu$ are position and momentum components, respectively? (5%) (b)From the commutation relations for the component of $\widehat{r}$ with those of $\widehat{p}$ ($[\widehat{x}_\mu, \widehat{p}_\nu] = i\hbar\delta_{\mu\nu}$), find all the commutation relations among the angular momentum operator $\widehat{L}_x$, $\widehat{L}_y$, $\widehat{L}_z$, and $\widehat{L}^2$, where $\overrightarrow{L} = \overrightarrow{r} \times \overrightarrow{p}$ in the classical mechanics. (5%) (b) By the introduction of the Hermitian conjugate pair of operators $\widehat{L}_+ = \widehat{L}_x + i\widehat{L}_y$ and $\widehat{L}_- = \widehat{L}_x - i\widehat{L}_y$, show that $\widehat{L}_\pm\psi_{l,m} = C_\pm(l,m)\psi_{l,m\pm1}$, where $C_\pm(l,m) = \hbar\sqrt{(l\mp m)(l\pm m+1)}$ and $\psi_{l,m}$ are the eigenfunctions of $\widehat{L}_2$ and $\widehat{L}_z$, respectively ($\widehat{L}^2\psi_{l,m} = \hbar^2 l(l+1)\psi_{l,m}$, $\widehat{L}_z\psi_{l,m} = \hbar m\psi_{l,m}$). (5%)

7. (10%) Two identical spin-1/2 particles of mass m move independently in a one-dimensional box of length L. Let $\phi_n(x)$ $(n = 1, 2, 3, \cdots)$ denote the single-particle spatial wave functions and $\chi_+(\chi_-)$ denotes the spin-up (down) state. (a) What is the two-particle total wave function of the lowest total energy of the system? (5%) (b) Write explicit expression for the expectation value of the separation distance D between the particles in the case of one particle being in the ground state and the other particle being in the first excited state. Will the expectation value affected by an exchange of the particle labels? (5%)

8. (10%) Explain briefly the following terms:

(a) diamagnetism, paramagnetism, and ferromagnetism (2%)

國立清華大學命題紙

八十七學年度 物理 系(所) ______ 組碩士班研究生入學考試

科目 近代物理 科號 0401 共 3 頁第 3 頁 *請在試卷【答案卷】內作答

(b) boson and fermion (2%)
(c) Hermitian operator (2%)
(d) the Zeeman effect (2%)
(e) the photoelectric effect (2%)

Physical Constants:
c (velocity of light)= 3.00×10^{8} m/sec
ϵ_0 (vacuum permittivity)= 8.85×10^{-12} coul /V·m
h (Plank constant)= 6.63×10^{-34} J·sec= 4.14×10^{-15} eV·sec
$\hbar = h/2\pi = 1.06 \times 10^{-34}$ J·sec= 6.58×10^{-16} eV·sec
m_e (electron mass)= 9.11×10^{-31} kg
e (electron charge)= 1.60×10^{-19} coul
α (fine structure constant)= $\frac{e^2}{4\pi\epsilon_0 \hbar c} \cong \frac{1}{137}$
$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{e^2 m_e} = 5.29 \times 10^{-11}$ m
Eigenenergies and eigenfunctions of 1-D simple harmonic oscillator:
$\widehat{H} = -(\frac{\hbar^2}{2m})\frac{d^2}{dx^2} + \frac{1}{2}m\varpi^2 x^2$
$E_n = (n + \frac{1}{2})\hbar\varpi$
$\psi_n(x) = (\frac{\alpha}{\sqrt{\pi}2^n n!})^{1/2} H_n(\alpha x)\exp(-\frac{1}{2}\alpha^2 x^2)$, where $\alpha = \sqrt{\frac{m\varpi}{\hbar}}$ and
$H_0(y) = 1$,
$H_1(y) = 2y$,
$H_2(y) = 4y^2 - 2, \cdots$