1. (15%) A photon is scattered by an electron at rest. The frequency of the incident photon is $\nu = \frac{m_e c^2}{h}$, where m_e is the rest mass of electron, c the speed of light, h the Planck constant. Assume the scattering angle of the scattered photon is 90° and its wavelength increases by $\frac{h}{m_e c}$.
   (a) Find the speed of the electron after scattering.
   (b) Find the angle between the scattering electron and the incident photon.
   (c) If the frequency of the incident photon keeps increasing, what are the limits of the answers to (a) and (b)?

2. (10%) A typical experiment to detect proton decay is to construct a very large reservoir of water and put into it devices to detect Cerenkov radiation produced by the products of proton decay.
   (a) Suppose that you have built a reservoir with 10,000 metric tons of water. If the proton mean life τ_p is 10^{23} years, how many decays would you expect to observe in one year? Assume that your detector is 100% efficient and the proton bound in nuclei and free proton decay at the same rate.
   (b) A possible proton decay is $p \rightarrow \pi^o + e^+$. The neutral pion π^o immediately (in 10^{-16} seconds) decays to two photons, $\pi^o \rightarrow \gamma + \gamma$. Calculate the maximum and minimum photon energies to be expected from proton decay at rest. (proton mass = 938 MeV/c^2; positron mass = 0.51 MeV/c^2; π^o mass = 135 MeV/c^2)

3. (10%) Carbon dioxide in the atmosphere contains a nearly steady state concentration of ^{14}C, which is continually produced by secondary cosmic rays interacting with atmospheric nitrogen. When a living organism dies, its carbon contains ^{14}C decreases due to radioactive decay. This is the basis for the technique of radiocarbon dating. In the following we assume that the atmospheric value for the ratio $^{14}C/^{12}C$ is 10^{-12} and that the half-life for the ^{14}C β^- decay is about 5500 years.
   (a) It is desired to determine the age of a carbon sample. How many grams of a sample are needed to measure the age to a precision of $\pm$ 60 years (standard deviation of 60 years)? Assume that the sample is actually 6000 years old, that the radioactivity is counted for one hour with a 100% efficient detector, and that there is no background.
   (b) Repeat part (a), but now assume that there is a background counting rate in the detector (due to radioactivity in the detector itself, cosmic ray, etc.) whose average value is accurately known to be 4000 counts per hour.

4. (25%) A spinless particle of mass m and charge q is constrained to move on a circle of radius R. Find its allowed energy levels (up to a common additive constant) for each of the following cases:
(a) the motion of the particle is non-relativistic,
(b) there is a uniform magnetic field **B** perpendicular to the plane of the circle;
(c) the same magnetic flux which passed through the circle is now contained in a solenoid of radius $b < R$;
(d) there is a very strong electric field **E** in the plane of the circle ($qE >> \hbar^2/mR^2$);
(e) B and E are zero, but the electron's motion around the circle is extremely relativistic.

5. (10%) A particle of mass m moves in the logarithmic potential $V(r) = C \ln r/r_o$.

Show that: (a) All eigenstates have the same mean square velocity, and find this mean square velocity. (b) The spacing between any two levels is independent of the mass m

6. (30%) Discuss briefly what was learned from the experiments listed in the following. You need not describe the experiments in detail – you should just make clear in what ways our knowledge and understanding of the physical world improved as the result of the experiment.
(a) The Davisson – Germer experiment
(b) Stern – Gerlach experiment
(c) The Zeeman effect discovery
(d) The observation of Compton scattering
(e) The observation of a correlation between electron and spin in the beta decay of polarized Co^{60} by Madame Wu
(f) The observation of decays of the long-lived K^0 meson (K_L^0) into two pions by Fitch and Cronin.

Physical constants:
$c = 3.0 \times 10^8$ m/sec; h (Planck constant) $= 6.63 \times 10^{-34}$ J.sec $= 4.14 \times 10^{-15}$ eV.sec
m_e (electron mass) $= 9.11 \times 10^{-31}$ kg; e (electron charge) $= 1.60 \times 10^{-19}$ C.