

1. (10%) Find the sum of the infinite series

$$1 + 2x + 3x^2 + 4x^3 + \dots + (n+1)x^n + \dots$$

where $|x| < 1$.

2. (10%) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 4 \\ 2 & 10 \end{bmatrix}$$

3. (20%) Solve

$$\frac{d^4 y}{dx^4} + 2\frac{d^2 y}{dx^2} + y = 0.$$

4. (20%) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

5. (20%) Expand the function $f(x) = x$, $-2 < x < 2$ in a Fourier series.

6. (20%) Evaluate the integral

$$\int_0^{2\pi} \frac{1}{(2 + \cos \theta)^2} d\theta.$$