

NTHU physics Entrance Exam Modern Physics

Useful information

$$\begin{aligned} h &= 6.63 \times 10^{-34} \text{ joule} \cdot \text{sec} \\ \text{electron rest mass} &= 9.109 \times 10^{-31} \text{ kg} \\ e &= 1.6 \times 10^{-19} \text{ Coulomb} \end{aligned}$$

Problem 1 (20%) Indicate briefly the most important physics that was demonstrated in the following experiments/phenomenon

- (a) Compton effect
- (b) Young's Two-Slit interference experiment
- (c) Davisson-Germer experiment
- (d) Stern-Gerlach experiment
- (e) Franck-Hertz experiment.

Problem 2 (10%) Consider the radiation in a three dimensional cavity. Let us denote $\rho(\nu)$ as the equilibrium energy density of the radiation per volume and per frequency.

- (a) (5%) If we now dig a small hole with area A on the surface of the cavity, find the total power radiated out of the hole in terms of $\rho(\nu)$ (assuming that radiation incident on the hole has no reflection at all).
- (b) (5%) If we put atoms with two energy levels ϵ_0 (ground state) and ϵ_1 (excited state) in the cavity to interact with the radiation and assume that the temperature is T , find the ratio of the probability for the spontaneous emission to that for the stimulated emission for transition from ϵ_1 to ϵ_0 .

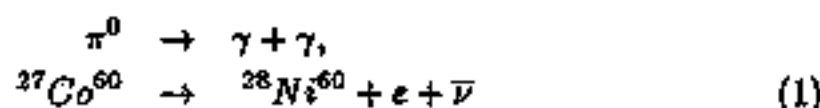
Problem 3 (10%)

- (a) (5%) Write out the configurations (for electronic orbits) for the ground states of ^{29}Cu and ^{23}V .
- (b) (5%) How many lines would be expected on the detector plane of a Stern-Gerlach experiment if we use a beam of atoms with the state $^3\text{D}_2$? (assuming that the external field is strong enough to destroy the coupling between L and S) Indicate which line has the strongest intensity.

Problem 4 (10%)

- (a) (5%) Write down the fundamental interactions of the nature and specify the names of the particles that mediate these interactions. If the mass of the mediate particle for an interaction is m_0 , estimate the range of that interaction.

- (b) (5%) Indicate the most important fundamental interactions involved in the following reactions:



Problem 5 (15%) Consider the three-dimensional harmonic trap for Bose-Einstein condensate. The Hamiltonian is

$$H = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + \frac{1}{2}m\omega^2(x^2 + y^2 + z^2). \quad (2)$$

- (a) (5%) Show that the Hamiltonian commutes with the x -component of the angular momentum

$$[H, L_x] = 0. \quad (3)$$

Make use of the permutation symmetry between x , y and z to show the Hamiltonian also commutes with the other two components,

$$[H, L_y] = 0, \quad [H, L_z] = 0. \quad (4)$$

- (b) (5%) Since L_x, L_y, L_z all commute with the Hamiltonian, can we use all l_x, l_y, l_z as good quantum numbers to label the eigenstates? Why or Why not?
- (c) (5%) Near zero temperature, bosons are trapped in the ground state. What is the rough length scale of the boson cluster?

Problem 6 (15%)

- (a) (5%) The wave function of a particle at $t = 0$ is $|\psi(0)\rangle = |E\rangle$, where $|E\rangle$ is an eigenstate with energy E . As time evolves, the wave function $|\psi(t)\rangle$ also varies with respect to time. What is the time derivative of the expectation of an arbitrary operator \hat{O} at later time t ,

$$\frac{d\langle \hat{O} \rangle}{dt} \equiv \frac{d}{dt} \langle \psi(t) | \hat{O} | \psi(t) \rangle = ? \quad (5)$$

- (b) (10%) Suppose the initial wave function at $t = 0$ is $|\psi(0)\rangle = (1/\sqrt{2})|E_1\rangle + (1/\sqrt{2})|E_2\rangle$. The matrix elements of the operator \hat{O} are $\langle E_1 | \hat{O} | E_1 \rangle = \langle E_2 | \hat{O} | E_2 \rangle = O_d$ and $\langle E_1 | \hat{O} | E_2 \rangle = \langle E_2 | \hat{O} | E_1 \rangle = O_t$, where both O_d and O_t are real numbers. Show that

$$\frac{d\langle \hat{O} \rangle}{dt} = A \sin(\omega t), \quad (6)$$

and find out A and ω explicitly.

Problem 7 (20%) Consider a one-dimensional wire with a delta-function scatterer at the origin. The Hamiltonian is

$$H = \frac{p^2}{2m} + \frac{\hbar^2}{ma} \delta(x). \quad (7)$$

A right-moving wave with wave number $k > 0$ can be cast into the form

$$\psi(x) = \begin{cases} e^{ikx} + re^{-ikx}, & x \leq 0, \\ te^{ikx}, & x \geq 0. \end{cases} \quad (8)$$

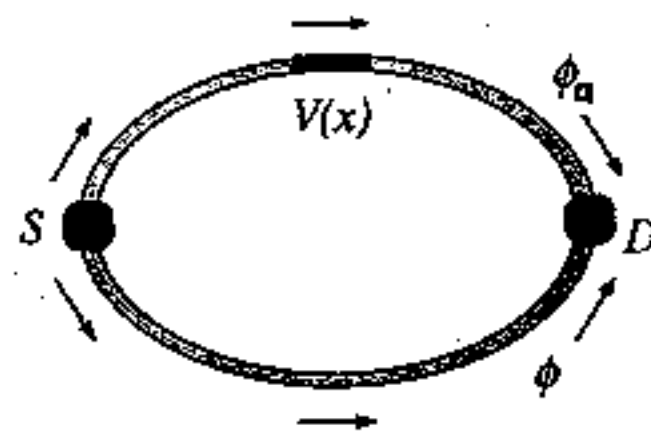


Figure 1: The interference between two quantum wires with a delta-function scatter at the upper wire.

- (10%) Find the reflection and transmission amplitudes r and t respectively.
- (5%) Calculate the current density $j(x)$ on both sides and verify the current is indeed conserved.
- (5%) Place a coherent source of particles at S and let particles travel through two different paths with equal length as shown in Fig. 1. Since there is a delta-function scatter at the upper path while the other path is completely free, quantum interference is expected. Calculate the phase difference $\Delta\phi = \phi_s - \phi$ at the destination D , where ϕ_s is the phase for the path with scatter and ϕ is the for the other path.