

國立清華大學命題紙

九十一學年度 物理、天文 系(所) _____ 組碩士班研究生招生考試

科目 應用數學 科號 0403 0502 共 一 頁第 _____ 頁 *請在試卷【答案卷】內作答

- (10 points) An unknown vector \mathbf{x} satisfies the relations: $\mathbf{x} \cdot \mathbf{b} = \beta$, and $\mathbf{x} \times \mathbf{b} = \mathbf{c}$. Try to express \mathbf{x} in terms of β , \mathbf{b} , and \mathbf{c} .
- (10 points) Construct the tangential plane passing through an arbitrary point $P(x_0, y_0, z_0)$ on an ellipsoidal surface given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

- (10 points) The axes of two circular cylinders of same radius a , intersect at right angles. Express the parametric equation for the intersection curve and calculate the common volume.
- (10 points) Let the matrix A be diagonalizable. Prove that

$$\det(\exp A) = \exp(\text{Tr} A),$$

and check it with $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

- (20 points) Solve the followings:
 - $yy'' = 2(y')^2 - 2y'$, $y(0) = 1$, and $y'(0) = 2$.
 - $y(t) = t + \int_0^t y(u) \sinh(t-u) du$.
- (20 points) Integrate the followings:
 - $\int_{-\infty}^{\infty} x \sin \pi x dx / (x^2 + 2x + 5)$,
 - $\int_0^{\infty} \sin^2 x dx / x^2$.
- (10 points) Use the function $f(x) = x^2$, $-\pi \leq x \leq \pi$
 - to express the Fourier series,
 - and to calculate the Riemann zeta functions: $\zeta_n = \sum_{p=1}^{\infty} \frac{1}{p^n}$, for $n = 2$, and 4.
- (10 points) Assuming that the temperature distribution function $u(x, t)$ of a uniform metal bar of length l satisfies

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{1}{c^2} \frac{\partial u(x, t)}{\partial t},$$

with constant c . Let the boundary and initial conditions be given as:

$$u(0, t) = u(l, t) = 0$$

$$u(x, 0) = \begin{cases} x, & 0 \leq x < l/2, \\ l-x, & l/2 < x \leq l. \end{cases}$$

Solve the temperature distribution function $u(x, t)$.