Useful physical quantities

$h = Plack$ constant $= 6.63 \cdot 10^{-34}\ J \cdot \sec = 4.14 \cdot 10^{-15}\ eV \sec$

$\hbar = \frac{h}{2\pi} = 1.05 \cdot 10^{-34}\ J \sec = 6.58 \cdot 10^{-16}\ eV \sec$

$M_{\pi^+} c^2 \approx 140\ MeV$

$c = 3 \cdot 10^8\ m/\sec,\ \hbar c = 197\ eV \cdot nm$

$\ell n2 = 0.693$

$m_e c^2 \cong 0.511\ MeV$

$1\,eV = 1.6 \cdot 10^{-19}\ J$

Pauli matrices:

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Part I (50%)

There are 10 blanks A to J in this part. Each gets 5 points (5%). Please list the answers of each blank on the first page of your answer sheet.

1. A group of π^+ mesons is observed travelling at speed 0.8c in a particle physics laboratory. If pion's (proper) lifetime is 1.8×10^{-8} sec. Their lifetime in the laboratory frame is __(A)__ . If there were initially 32,000 pions, the number of pions left, on the average, after travelling 36 m is __(B)__ .
2. Two rockets are leaving their space station along perpendicular paths, as measured by an observer on the space station. Rocket 1 moves (along x axis) at 0.6c and Rocket 2 moves (along y axis) at 0.8c, both measured relative to the space station. $V_x^{'}$, $V_y^{'}$ of Rocket 2 as observed by Rocket 1 are __(C)__ , __(D)__ respectively.
3. The radius of the circular trajectory of an π^+ in a magnetic field of 1 T is 0.96 m. The kinetic energy of the π^+ is __(E)__ .
4. What kinetic energy (in electron volts) __(F)__ should neutrons have if they are to be defracted from crystal? (Assume the interatomic distance in crystal is approximately 1 Å)
5. Given $[L_x, L_y] = i\hbar L_z, [L_y, L_z] = i\hbar L_x, [L_z, L_x] = i\hbar L_y$ and define $L_\pm = L_x \pm iL_y$, then $[L_+, L_-]$ and $[L_z, L_\pm]$ are given by __(G)__ and __(H)__ respectively.
6. If a system is described by the Hamiltonian $H = \frac{\vec{L}^2}{2I} + \alpha L_z$, then the energy eigenvalues of the system are given by __(I)__ with corresponding eigenfunctions __(J)__ 。

Part II (50%)

There are four problems in this part. Please write down the answers start from the second page of your answer sheet

7. (15%) Suppose an electron (a spin 1/2 particle) is in the spin state

$\frac{1}{\sqrt{6}}\begin{pmatrix}1+i\\2\end{pmatrix}$ at time $t=0$

(1) If one measures S_z at t=0, what is the probability of getting $\frac{\hbar}{2}$?

If one measures S_x at t=0, what is the probability of getting $-\frac{\hbar}{2}$?

Electron has magnetic moment $\vec{\mu}=\gamma\vec{S}$ (γ is the gyromagnetic ratio)

If the electron is at rest in a magnetic field, along the z direction $\vec{B}=B_0\hat{k}$

(2) With the initial condition given above, find the spin state at time t.

(3) If S_z is measured at time t, what is the probability of getting $\hbar/2$?

If S_y is measured at time t, what is the probability of getting $-\hbar/2$.

8. (8%) The bottom of an infinite well is changed to have the shape $V(x)=\varepsilon\sin\frac{\pi x}{b}$ for $0\le x\le b$

Calculate the energy shifts for all the excited state to first order in ε.

[Note that the well originally had $V(x)=0$ for $0\le x\le b$ with $V=\infty$ elsewhere]

9. (7%) (1) Which is the lowest energy of a set of 24 noninteracting electrons in a cubic box of length L?

Given $E_{n_x,n_y,n_z}=\frac{\hbar^2\pi^2}{2ML^3}(n_x^2+n_y^2+n_z^2)$ with (n_x, n_y, n_z) being 1, 2, 3....

(2) What would be the lowest energy of set of 24 identical bosons in a cubic box of length L?

10. (20%) Give a concise definition/explanation of following terms:

(1) Laser cooling, Bose-Einstein condensation.

(2) Light-emitting diode.

(3) BCS theory

(4) $W^{\pm}$, Z particles, Gluon.

(5) Dark matter, Dark energy

Example:

Fusion：The nuclear process whereby two light nuclei can overcome the mutual electric (Coulomb) repulsion to fuse together. This is accompanied by the released of a large amount of energy and is the source of energy in the sun and other stars. It is hoped that fusion will one day be harnessed as an energy source on Earth.