

10% 1. Evaluate $\lim_{x \rightarrow \infty} (\ln x)^{\frac{1}{x}}$.

10% 2. Evaluate $\int x \cos x dx$

10% 3. Evaluate $\lim_{x \rightarrow \infty} \frac{1}{x \ln x} \int_1^x \ln t dt$.

10% 4. Does the series $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$ converge? Give your reason.

10% 5. Apply the double integrals to compute $\int_0^{\infty} e^{-x^2} dx$.

20% 6. Prove the following matrix A is similar (over \mathbb{R}) to a diagonal matrix

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

18% 7. V is a finite dimensional vector space and $f: V \rightarrow V$ is a linear transformation.

Prove the following statements are equivalent

(a) f is injective

(b) f is surjective

(c) f is an isomorphism

12% 8. Find the minimal polynomial (over \mathbb{R}) of the following matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$