

國立清華大學 命題紙

九十二學年度 統計學研究所 (所) 組碩士班研究生招生考試

科目 機率論 科號 0302 共 2 頁第 1 頁 \*請在試卷【答案卷】內作答

- 20% 1. Write down the probability function, the mean, the variance and the moment generating function of each of the following distributions:  
Binomial, Poisson, Chi square and normal.
- 20% 2. Let  $X, X_1, X_2, \dots, X_n, \dots$  be random variables. State the definition of the various versions of the convergence that  $X_n \rightarrow X$  as  $n \rightarrow \infty$ .
- 10% 3. Let  $X$  be a random variable. Show that  $E(X - a)^2$  is minimized at  $a = E(X)$  and that  $E|X - a|$  is minimized at  $a = \text{median of } X$ .
- 10% 4. Choose two points on the unit circle randomly. Find the probability density of the length of the chord connecting the two points.
- 10% 5. A box contains balls numbered 1 to  $N$ . Let  $X$  be the largest number drawn in  $n$  drawings when random sampling with replacement is used. Compute  $E(X)$  and show that it can be approximated by  $nN/(n+1)$ .
- 15% 6. Toss a fair coin and take a step of size  $c$  to the right if head shows, to the left if tail shows. At the  $n$ th toss, the position is a random variable  $X_n$  taking the values  $mc$ , where  $m = n, n-2, \dots, -n$ .
- a. Find  $P(X_n = mc)$
- b. Show that for large  $n$

$$P(X_n = mc) \approx \frac{1}{\sqrt{n\pi/2}} e^{-m^2/2n}$$

- c. Find the probability that at the 50<sup>th</sup> step,  $X_{50}$  will exceed  $6c$ .

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- 15% 7. A miner is trapped in a mine containing 3 doors. The first door leads to a tunnel that will take him to safety after 3 hours of travel. The second door leads to a tunnel that will return him to the mine after 5 hours of travel. The third door leads to a tunnel that will return him to the mine after 7 hours. Assume that the miner is, at all time, equally likely to choose any one of the doors, what is the expected length of time he reaches safety?