

九十三學年度 統 計 (所) \_\_\_\_\_ 組碩士班入學考試

科目 機 率 論 科號 0302 共 2 頁第 1 頁 \*請在試卷【答案卷】內作答

A 填充題 (共 10 題, 每題 8 分) 請務必將答案寫在 答案卷的第一頁,  
並清楚標示題號, 解題的計算過程不列入評分。

1. On the expansion of  $(x_1 + x_2 + \cdots + x_{10})^{100}$ , let  $A$  be the number of terms in the expansion, and  $B$  be the coefficient of term  $x_1^{50} x_{10}^{50}$ . Then  $(A, B) =$  \_\_\_\_\_.
2. On a line of length  $2L$ , a point is randomly selected from the first half, and then independently another point is randomly selected from the second half. What is the probability that the distance between these two points is greater than  $L/3$ ?  
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3. From all the possible subsets (including empty subset) of a set of  $N$  elements, two subsets  $A, B$  are randomly and independently selected. Then  $\Pr\{A \subset B\} =$  \_\_\_\_\_.
4. Supposed that engines in an airplane function independently with probability  $p$ ,  $0 < p < 1$ , and that an airplane needs a majority of its engines functioning to make a flight. For what values of  $p$  is a 5-engine plane is preferable to a 3-engine plane? \_\_\_\_\_.
5. Ten persons out of 100 married couples are randomly chosen to answer a questionnaire. What is the expected number of couples that both are not chosen to answer the questionnaire? \_\_\_\_\_
6. Let  $X$  is distributed as  $Binomial(1, p)$ ,  $Y = 2X - 1$ . Suppose that  $c > 0$  be a constant such that  $E(c^Y) = 1$ . Then  $c = 1$  or  $c =$  \_\_\_\_\_.
7. Let  $X$  is distributed as  $Pisson(\lambda)$ ,  $\lambda > 0$ . Then  
$$\sum_{n=0}^{\infty} n \Pr\{X > n\} =$$
 \_\_\_\_\_.
8. Let the joint density function of the random variables  $X, Y$  be  
$$f(x, y) = \frac{e^{-y}}{y}, \quad 0 < x < y, \quad 0 < y < \infty.$$
  
Let  $A = E(X^2 | Y = 10)$ ,  $B = Cov(X, Y)$ . Then  $A + B =$  \_\_\_\_\_.

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9. Let  $X, Y$  be integer-valued random variables with joint probability mass function

$$f(x, y) = \begin{cases} \frac{2}{n(n+1)}, & y = 1, \dots, x, x = 1, \dots, n \\ 0, & \text{otherwise} \end{cases}$$

Then  $E(X | Y = y) + E(Y | X = x) =$  \_\_\_\_\_.

10. Let  $X_1, X_2, X_3$  be independent and identically distributed with density

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

Set  $Y_1 = X_1 / (X_2 + X_3)$ ,  $Y_2 = (X_1 + X_2) / (X_1 + X_2 + X_3)$ ,  $Y_3 = X_1 + X_2 + X_3$ .

Then  $E(Y_1 + Y_2 + Y_3) =$  \_\_\_\_\_.

B. 證明題 (共 1 題, 20 分, 請從答案卷第二頁寫起, 第一頁留給填充題)

1. Random variable  $X$  is called stochastically larger than random variable  $Y$  if and only if  $\Pr\{X > t\} \geq \Pr\{Y > t\}$  for every real number  $t$ . Show that  $X$  is stochastically larger than  $Y$  if and only if  $Ef(X) \geq Ef(Y)$  for all increasing function  $f$ .