94學年度 統 計 (所) ________ 組碩士班入學考試

科目 基礎數學 科目代碼 0301 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

作答時，請非常清楚地標示各題號。非証明題(問題1–4)之解題或計算過程不列入評分。

1. (10%) The general solution of the equation $y''+9y=\sin 3x$ is ________.

2. (7%) Let $f(x)=\sum_{n=0}^{\infty} a_n x^n$, where the coefficients a_n are determined by the relation $\cos x=\sum_{n=0}^{\infty} a_n (n+2)x^n$. Then $f(\pi)=$ ________.

3. (7%) A linear Cartesian equation for the plane through (2, 3, 1) parallel to the plane through the origin spanned by (2, 0, -2) and (1, 1, 1) is ________.

4. (a) (6%) $\lim_{n\to\infty} \frac{(n!)^2 2^{2n}}{(2n)!\sqrt{n}} =$ ________.

   (b) (6%) The function $f(x)=$ ________ is a non-zero continuous function satisfying $f^2(x)=\int_0^x f(t)\frac{\sin t}{2+\cos t}dt$.

   (c) (6%) If a is an arbitrary real number, let $s_n(a)=1^a+2^2+\ldots+n^a$. Then $\lim_{n\to\infty}\frac{s_n(a+1)}{n s_n(a)}=$ ________.

   (d) (6%) $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2=$ ________.

5. (6%) Prove that $\int_{-\infty}^{\infty} f(x)dx=1$, where $f(x)=(a\sqrt{2\pi})^{-1}\exp\{-\frac{1}{2}(x-b)^2/a^2\}$ with $a>0$.

6. (10%) Let $f(x,y)=k_1\cdot\exp\{-k_2[x^2-2\rho xy+y^2]\}$ with positive constants k_1 and k_2 and $-1<\rho<1$, where $-\infty<x,y<\infty$. Characterize the set of all (x,y)'s such that $f(x,y)=c$, where c is a given constant satisfying $0<c<\underset{x,y}{\text{Max}}\, f(x,y)$. In other words, give characteristics that will uniquely determine the graph of the set.

7. Let $f(x_1,x_2,x_3)=4x_1^2+4x_2^2+4x_3^2+4x_1x_2+4x_1x_3+4x_2x_3$, $-\infty<x_1,x_2,x_3<\infty$.
   (a) (5%) Describe the set of all (x_1,x_2,x_3)'s such that $f(x_1,x_2,x_3)\geq 0$.
   (b) (7%) Find a linear operator, in terms of a matrix, to transform the given coordinate system to a new coordinate system so the transformed function of f will not contain any cross-product terms.

8. (10%) Find the maximum of $f(x_1,x_2,x_3)=5x_1+6x_2+7x_3$, subject to the following constraints: $x_1+2x_2+3x_3=11$, $3x_1+x_2+x_3=10$, and $x_1+4x_2+x_3\leq 15$.

9. (7%) Let $\mathbf{X}=[X_{\alpha\beta}]$ be a $p\times n$ data matrix, where $X_{\alpha\beta}$ is the β-th observation on the α-th variable. Define $\boldsymbol{\varepsilon}=(1,1,\ldots,1)'\in R^n$, which determines an equiangular line. Consider the ith and jth rows, $\mathbf{x}_i'$ and $\mathbf{x}_j'$, of $\mathbf{X}$, and let $\mathbf{u}_i$ and $\mathbf{u}_j$ be the corresponding projections on $\boldsymbol{\varepsilon}$. Is there any statistical interpretation of the cosine of the angle between $\mathbf{x}_i-\mathbf{u}_i$ and $\mathbf{x}_j-\mathbf{u}_j$?

10. (7%) Assume that $\mathbf{M}=\begin{pmatrix}0.9 & 0.2\\ 0.1 & 0.8\end{pmatrix}$. Without finding $\mathbf{M}^{-1}$ explicitly, compute $\mathbf{M}^{-25}$.