

國立清華大學命題紙

95 學年度 統計 所碩士班入學考試

科目 基礎數學 科目代碼 0301 共 3 頁第 1 頁 *請在【答案卷卡】內作答

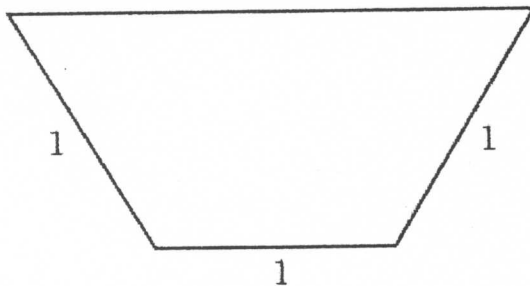
共 12 題

1. (5%) (a) find the equation of the tangent line to the graph of the equation $x^2 + 4y^2 = 16$ at the point $(-2, -\sqrt{3})$

(5%) (b) Suppose that $\ln(x^2 + y^2) = x^3 + y^2$. Find the slope of a line tangent to the graph of this curve at any point on the curve.

2. (5%) 有個小男孩買了一個最多可以吹到 1 立方英尺的氣球，他以每分鐘增加 0.1 立方英尺的速度吹氣進去，當氣球達到 0.5 立方英尺大小時，氣球半徑增加的瞬間速度為何？（假設氣球在任何時刻都是正球形並且吹氣速度均勻）

3. (5%) Three of the sides of a trapezoid have length 1. What should be the length of the fourth side if the area of the trapezoid is to be as large as possible?



4. (7%) If $0 \leq f(x)$ and $\lim_{x \rightarrow a} f(x) = 0$, and $\lim_{x \rightarrow a} g(x) = +\infty$, $\lim_{x \rightarrow a} f(x)^{g(x)} = ?$

5. (8%) (a) $\lim_{x \rightarrow \infty} [x \ln(1 + x^{-1})]^x = ?$

(5%) (b) $\lim_{x \rightarrow 0} \frac{xe^{3x}}{\tan \pi x} = ?$

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6. Determine whether the following are convergent or not.

(5%) (a) $\sum_{n \geq 1} \frac{\tan^{-1}(n)}{n^2 + 1}$,

(5%) (b) $\int_0^{\infty} \frac{1}{\sqrt{x+x^3}} dx$

7. (10%) Find a matrix A so the substitution $\begin{bmatrix} x \\ y \end{bmatrix} = A \begin{bmatrix} s \\ t \end{bmatrix}$ transforms the quadratic form $x^2 + 4xy + y^2$ into the quadratic form $s^2 - t^2$.

8. (10%) Find (s, t) such that $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$ is as close as possible to $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

9. (5%) Determine which of the following sets are subspaces of \mathbb{R}^n ?

(i) $A = \{(x_1, x_2, \dots, x_n) \in \mathbb{R}^n : x_1 = 0 \text{ or } x_n = 0\}$

(ii) $B = \{(x_1, x_2, \dots, x_n) \in \mathbb{R}^n : x_1 + 2x_n = 0\}$

(iii) $C = \{(x_1, x_2, \dots, x_n) \in \mathbb{R}^n : \sum_{i=1}^n x_i = 1\}$

10. (5%) (i) $S_1 = \{(1, 2, 3), (2, 3, 4)\}$,

(ii) $S_2 = \{(5, 8, 11), (1, 1, 1), (1, -1, -3)\}$.

Is the space spanned by the set S_1 a subspace of the space spanned by S_2 ?

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11. (10%) Let $\{e_1, e_2, \dots, e_n\}$ be the standard basis of \mathbb{R}^n , and $f: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear map given on the standard basis by $f(e_1) = 2e_2 + e_3$ and $f(e_2) = e_1 + 3e_2 + 5e_3$

Determine the 3×2 matrix of this map with respect to the bases

$3e_1 - e_2, 2e_1 + 3e_2$ of \mathbb{R}^2 and $e_1, e_1 + e_2, e_1 - e_2 - e_3$ of \mathbb{R}^3 .

12. Please decide whether the following questions are True or False.

(2%)(a) Let A and B be $m \times n$ matrices. If A is row-equivalent to B , then $\text{Col}(A) = \text{Col}(B)$

(2%)(b) Distinct eigenvectors are linearly independent.

(2%)(c) Let A be a real 2×2 matrix, whose characteristic polynomial does not have real roots. Then A is diagonalizable.

(2%)(d) If A is an $n \times n$ matrix with fewer than n distinct eigenvalues, then A is not diagonalizable.

(2%)(e) Every vector space is spanned by a linearly dependent set.