

八十八學年度 工程與系統科學系(所) 丁戊 組碩士班研究生招生考試

科目 電磁學 科號 ³⁶⁰²3503 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

**** Show your derivations in details !!**

Make clear all your assumptions/approximations!!

1. Consider two 2 cm radius circular loops which have the number of turns 5 and 10, respectively. One of the loop is located at the origin and oriented in the z-direction. Neglect the thickness of the wire and find their mutual inductance if the 2nd loop is : (10%)

- (a) placed at $z = 100 \text{ cm}$ and oriented in the z-direction. (Fig. 1 (a))
 (b) placed at $y = 20 \text{ cm}$ and oriented in the x-direction. (Fig. 1(b))

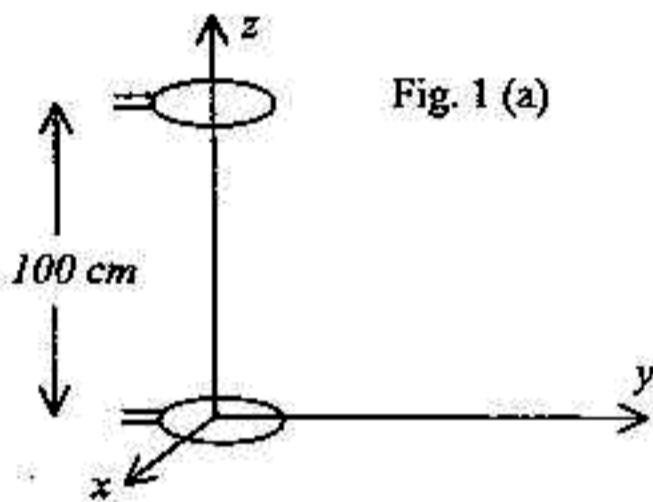


Fig. 1 (a)

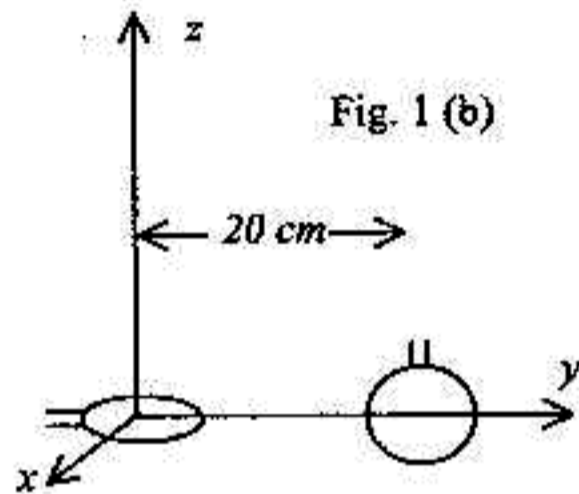


Fig. 1 (b)

2. A point charge q is placed at a distance d from the center of a spherical cavity which has an inner radius a and is located inside a conducting sphere of radius R . The distance between the centers of the two spheres is b , as shown in Fig.3. Find:
 (a) electric field distribution everywhere, and (b) the force on the charge.
 (Note: For convenience, you may chose different coordinates / origins for the expressions of the electric field in different regions.) (20%)

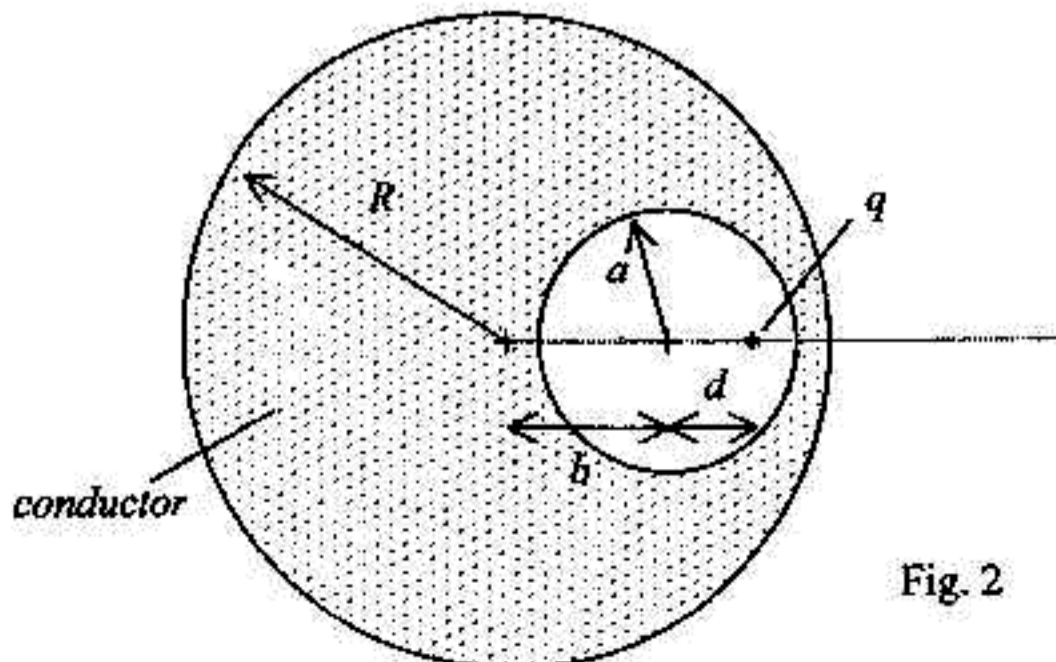


Fig. 2

3. Consider an infinite long rectangular pipe formed by perfect conductors and filled with a dielectric of permittivity ϵ , as illustrated in Fig. 2. One side of the pipe is held at electric potential V_0 and the other three are grounded. (20%)
- (a) Find the potential distribution inside the pipe.
- (b) Find the surface charge density distribution on the side held at potential V_0 .

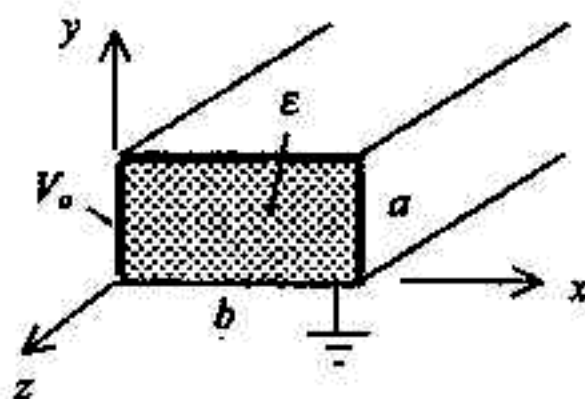


Fig. 3

4. A current transformer can be used to measure currents in a wire, as shown in Fig. 4. Assuming the infinite long wire has a diameter of 2 cm and coincides with the principle axis of symmetry of a 100 turn coil wrapped uniformly around a rectangular, toroidal-shaped iron core of inner and outer radii $a = 8$ cm and $b = 10$ cm, thickness $t = 2$ cm, and relative permeability $\mu_r = 1000$. The current, which is uniformly distributed in the wire, varies with time sinusoidally at frequency 400 Hz and amplitude 1 Ampere peak to peak. Find the induced voltage V_{ind} between the terminals of the coil. (15%)

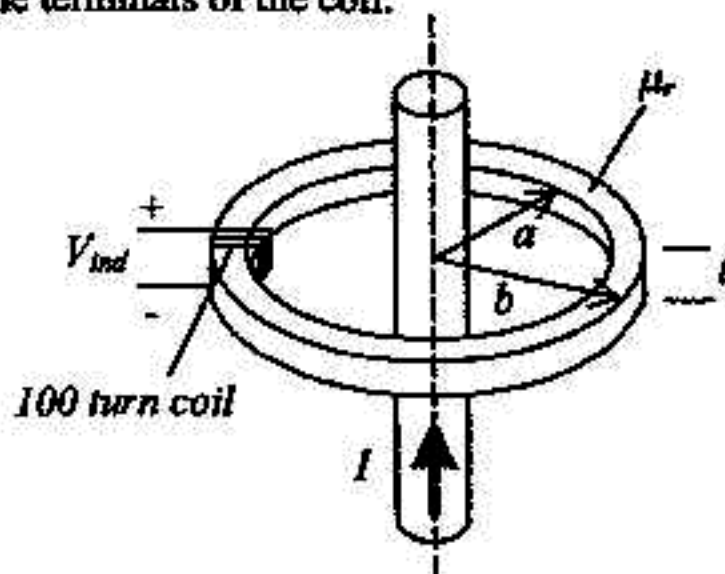


Fig. 4

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5. Consider a rectangular waveguide made of perfect conductor and filled with air, as shown in the Fig. 5,

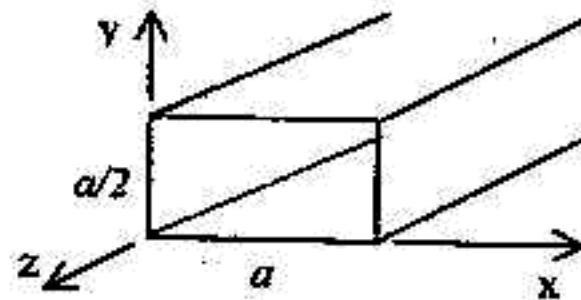


Fig. 5

- (a) What is the waveguide mode with the lowest cutoff frequency? Qualitatively plot its dispersion diagram ($\omega - k_z$ relation). (5%)
- (b) What are the waveguide modes having the 2nd lowest cutoff frequency? (5%)
- (c) Find the cutoff frequencies in (a) and (b) if $a = 10$ cm. (5%)
- (d) Find the *phase* and *group* velocities for waves of frequency 3 GHz. (5%)
6. In an experiment, one needs to transmit a monochromatic light of wavelength 550 nm from air into a vacuum chamber through a window made of BK-7 glass, which has an index of refraction of $n = 1.52$, as shown in Fig. 6. Propose all the methods you can think of such that the transmission is perfect, i.e., there is no reflection. Note that you need to PROVE your methods work by writing down the related equations and calculations. (Hint: You may choose the thickness or orientation of the glass window or any coating you may add to its surface, etc.) (15%)

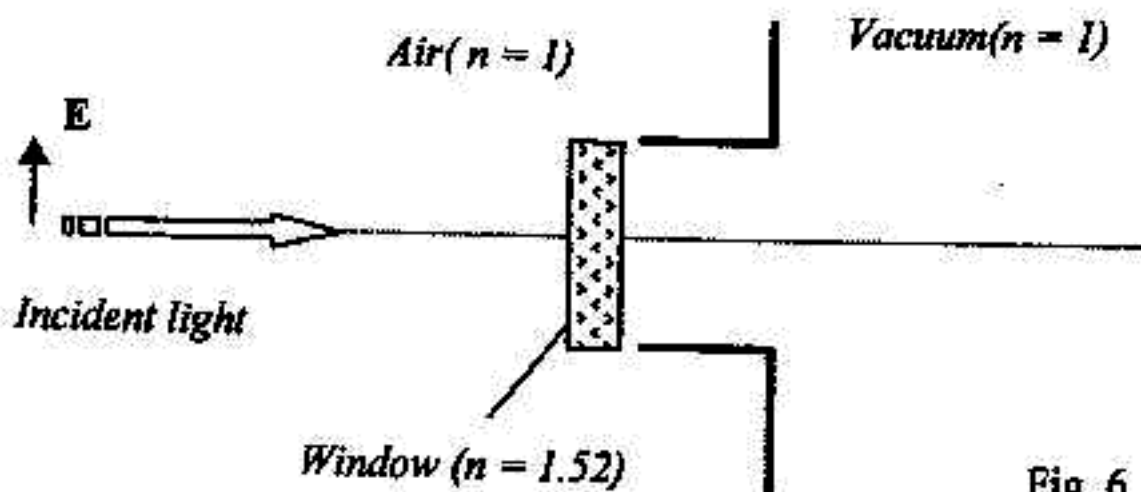


Fig. 6