

國立清華大學命題紙

八十七學年度 工程系統科學 系(所) \_\_\_\_\_ 組碩士班研究生入學考試

科目 工程數學 科號 4001 共 2 頁第 1 頁 \*請在試卷【答案卷】內作答

1. Find the general solution

$$y'' - 2y' + y(x) = 4 + xe^x; \quad y(0) = 1, y'(0) = 1. \quad (15\%)$$

2. Find the solution of the initial value problem

$$y'' + 2y' + 2y(t) = \cos t + \delta(t - \pi/2); \quad y(0) = 0, y'(0) = 0. \quad (20\%)$$

3. Evaluate the integral formula

$$\int_{-\infty}^{\infty} \frac{x \cos(x)}{x^2 - 3x + 2} dx.$$

Hint: Use residue theory. (15%)

八十七學年度 工程與系統科學系(所) 組碩士班研究生入學考試

科目 工程數學 科號 400/ 共 2 頁第 2 頁 \*請在試卷【答案卷】內作答

4. (15%) Use the Gauss' Divergence theorem to evaluate the integral

$$\int\int_S (x^2 + y + z) \, dS,$$

where  $S$  is the surface of the unit sphere  $x^2 + y^2 + z^2 = 1$ .

5. (20%) Consider a rectangular region  $0 \leq x \leq \pi$ ,  $-a \leq y \leq a$ . The two edges  $x = 0$  and  $x = \pi$  are maintained at zero temperature, and the other two edges are maintained at a constant value  $T_0$ . Find the steady-state temperature distribution, which satisfies the two-dimensional Laplace's equation, inside this enclosure.

6. (15%) The random variable  $X$ , representing the number of raisins in a raisin cup cake, has the following probability distribution:

$x$	4	5	6	7
$P(X = x)$	0.2	0.4	0.3	0.1

- (a) Find the mean  $\mu$  and the variance  $\sigma^2$  of  $X$ .
- (b) Find the mean  $\mu_{\bar{X}}$  and the variance  $\sigma_{\bar{X}}^2$  of the mean  $\bar{X}$  for random samples of 36 raisin cakes.
- (c) Find the probability that the average number of raisins in 36 raisin cakes will be less than 5.5? Explain which "Table" will you need to look up the answer?