國 立 清 華 大 學 命 題 紙

八十七學年度工程與系統科學系（所）＿＿＿＿組碩士班研究生入學考試
科目 電磁學 科號 4006 共 4 頁第 1 頁 *請在試卷【答案卷】內作答

1. (a) For electrostatic fields in an isotropic, linear medium with a permittivity ε and a volume charge density ρ, which one of the following statements is correct:
   (i) $\nabla \cdot \vec{D} = 0$, $\nabla \times \vec{E} = \rho/\varepsilon$, (ii) $\nabla \times \vec{E} = 0$, $\nabla \cdot \vec{D} = \rho$,
   (iii) $\nabla \times \vec{D} = 0$, $\nabla \cdot \vec{D} = \rho$, (iv) $\nabla \times \vec{E} = 0$, $\nabla \cdot \vec{E} = \rho$ (2 %)
   (b) from the results of (a), derive the boundary conditions at the interface of two different media having permittivities, ε_1 and ε_2, respectively. (8%)

2. A point charge, Q = $1\ \mu C$, is located at a distance $10.0\ cm$ from the center of a conducting sphere of radius $4.0\ cm$. What's the pressure (force per unit area) on the conductor at point P? (5 %)

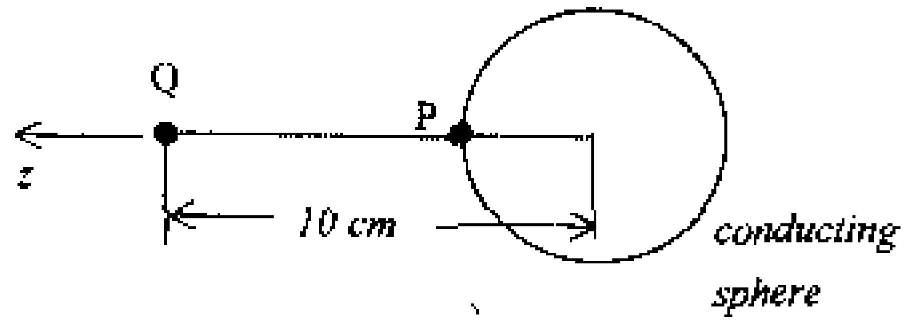

Fig. 1

3. A point charge Q is placed between a parallel plate capacitor with both plates grounded. If the separation of the plates is d and the distance of the charge from plate 1 is $d/3$ (distance from plate 2 is $2d/3$), find the induced charge on each plate. (10 %)

4. (a) Explain the Hall effect. (4 %)
   (a) For ferromagnetic materials, qualitatively sketch the hysteresis loops (B-H relation) and explain the physical origin of the hysteresis phenomena. (6 %)

5. A coaxial transmission line has a solid inner conductor of radius a, a very thin outer conductor of inner radius b. The region between the conductor is filled with a medium having constitutive parameters ($\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_0$). Determine its
   (a) capacitance per unit length. (5 %)
   (b) inductance per unit length. (5 %)

6. (a) A plane electromagnetic wave of frequency 1000 GHz is incident normally from air on a planar object formed by two layers of dielectric media having constitutive parameters, (ε_1, μ_1) and (ε_2, μ_2) respectively, as shown in Fig. 2. If $\varepsilon_1 = 4\varepsilon_0$ and $\mu_1 = \mu_2 = \mu_0$, find ε_2 and d such that there is no reflected wave. (5 %)

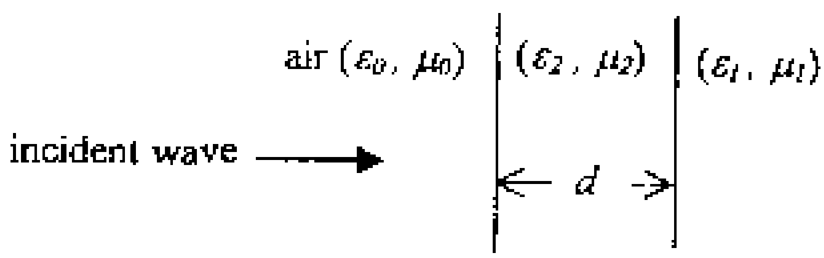

Fig. 2

(b) If $\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 = 4\varepsilon_0$ and $\mu_1 = \mu_2 = \mu_0$, determine the thickness d such that there is no reflection (3 %)

(c) A right-hand circularly polarized plane wave is incident normally on an infinite conducting plane. What's the polarization of the reflected wave? Explain your answer. (5 %)

7. For the **fundamental (or dominant) mode** of a rectangular waveguide made of perfect conductor and filled with air, as shown in the figure,

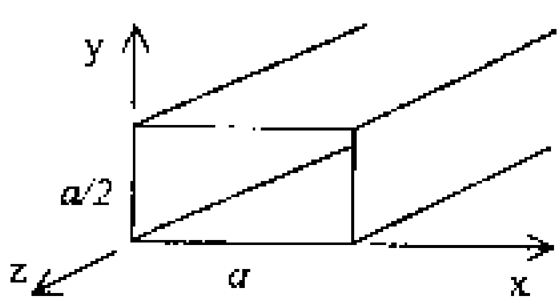

Fig. 3

(a) qualitatively plot the dispersion diagram (ω-k_z relation). (3 %)

(b) sketch the patterns of the transverse electric and magnetic fields on the waveguide cross section plane (x-y plane), and current density on the waveguide wall (top and right ones). (6 %)

(c) if $a = 5.0$ cm, find the cutoff frequency. (3 %)

(d) find the *phase* and *group* velocities for waves of frequency 5 GHz. (8 %)

8. (a) An electric dipole located at the origin oscillates at a frequency ω, $\vec{p} = \cos\omega t\,\hat{z}$. Qualitatively plot the *intensity profile* (i.e. radiation pattern) in the far field zone for the electromagnetic wave it radiates.
Repeat the above question for a magnetic dipole, $\vec{m} = \cos\omega t\,\hat{z}$. (6 %)
(note: A radiation pattern is a polar plot showing the dependence of the radiation intensity on the polar angle measured from the z-axis.)

(b) A loop antenna is placed at the origin and oriented along the z-direction (see Fig. 4). A 2nd dipole antenna far away is used to receive the signal transmitted from the loop antenna. What would be the orientation of the dipole antenna in order to maximize the received signal level if the dipole antenna is placed on *yz*-plane and an angle:
(i) $\theta = 0^{\circ}$ (ii) $\theta = 45^{\circ}$ (iii) $\theta = 90^{\circ}$ from the *z*-axis. (6 %)

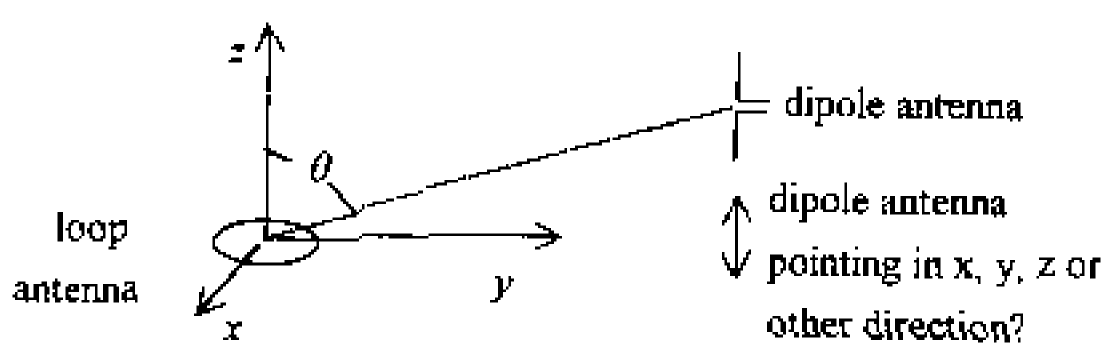

Fig. 4

9. A spherical balloon centered at the origin has a charge of amount Q uniformly distributed on its outer surface. Suppose the radius of the balloon, a, oscillates about a mean radius a_0 at a frequency ω according to, $a(t) = a_0(1 + 0.1\cos\omega t)$, find the electromagnetic fields at a distance $d = 1000\,a_0$ from the origin. (10 %)

Cylindrical Coordinates (r, ϕ, z)

$$\nabla V = \mathbf{a}_r \frac{\partial V}{\partial r} + \mathbf{a}_\phi \frac{\partial V}{r\partial \phi} + \mathbf{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial A_\phi}{r\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \mathbf{a}_r & \mathbf{a}_\phi r & \mathbf{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \mathbf{a}_r\left(\frac{\partial A_z}{r\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \mathbf{a}_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{a}_z\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (R, θ, ϕ)

$$\nabla V = \mathbf{a}_R \frac{\partial V}{\partial R} + \mathbf{a}_\theta \frac{\partial V}{R\partial \theta} + \mathbf{a}_\phi \frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix} = \mathbf{a}_R\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right]$$

$$+ \mathbf{a}_\theta\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right]$$

$$+ \mathbf{a}_\phi\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$