

八十六學年度 工程系統科學系(所) 組碩士班研究生入學考試

科目 工程數學 科號 4001 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Assume that the equation $f(x,y,z)=0$ defines x as a differentiable function of y and z , y as a differentiable function of x and z , and z as a differentiable function of x and y . f is differentiable and

$$f_x \cdot f_y \cdot f_z \neq 0. \text{ Find the value of } \left(\frac{\partial x}{\partial y}\right) \cdot \left(\frac{\partial y}{\partial z}\right) \cdot \left(\frac{\partial z}{\partial x}\right)$$

(15%)

2. Evaluate the vector integral

$$\frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\vec{R} \cdot \vec{r}} / |\vec{r}|^2 d^3 \vec{r}$$

where vectors $\vec{R} = (a, b, c)$ and $\vec{r} = (x, y, z)$

(15%)

3. Diagonalize the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

i.e. determine matrices Q and D , such the $Q^{-1}AQ=D$ is diagonal.

(15%)

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4. i) Find the Laplace transform of function

$$f(t) = \frac{\sin t}{t} \quad t > 0 \quad (10\%)$$

ii) Find the Laplace inversion of function

$$f(s) = \frac{1}{(s^2 + a^2)(s^2 + b^2)} \quad a, b \text{ are real parameters.} \quad (10\%)$$

5. Let C denote the unit circle $|z|=1$ taken counterclock wise.

Show that

$$\frac{1}{2\pi i} \oint_C \exp\left(z + \frac{1}{z}\right) dz = \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!} \quad (0! = 1) \quad (15\%)$$

6. Find the solution of the nonlinear ordinary differential equation

$$y''(x) + \sin y(x) = 0$$

with initial conditions : $y(0)=0, \quad y'(0)=2 \quad (20\%)$