

八十五學年度 材 = 材 = 材 (所) 組碩士班研究生入學考試

科目 工程數學 科號 3801 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Evaluate the following integrals

i) $\int_0^{\infty} \frac{\sin x}{x} dx$ (10%)

ii) $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$ (10%)

[Hint: i) Let $I(\lambda) = \int_0^{\infty} \frac{\sin x}{x} e^{-\lambda x} dx$, $\lambda > 0$, obtain $I'(\lambda)$.]

2. Evaluate the surface integral

$\int_S \underline{F} \times d\underline{\sigma}$ (15%)

where $\underline{F} = (x+y)\hat{z}$, and surface S: $x^2 + y^2 + z^2 = a^2$, $z \geq 0$

3. Consider the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

i) Find the eigenvalues and the eigenvectors of A. (10%)

ii) Calculate $\sin A$. (5%)

八十五學年度 物理系 (所) 組碩士班 研究生入學考試

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4. Solve the following ordinary differential equation

$$y'(x) + \omega^2 y(x) = F H(x - x_0) \quad \omega \text{ and } F \text{ are constants}$$

where $H(x)$ denotes the Heaviside step function, i. e., $H(x) = 1, x > 0$;
 $H(x) = 0, x < 0$, with initial conditions $y(0) = y'(0) = 0$.

(15%)

5. Obtain the first three non-zero terms of the Laurent expansion of $\csc z$ in the interval $0 < |z| < \pi$.

(15%)

6. Find the solution of wave equation

$$\partial^2 u / \partial t^2 - \partial^2 u / \partial x^2 + xt = 0 \quad |x| < \infty, t > 0$$

with initial conditions $u(x, 0) = \frac{\partial u}{\partial t}(x, 0) = 0$.

(20%)

(Hint: Change variables: $\xi = x + t, \eta = x - t$)