國立清華大學命題紙

八十四學年度核子工程與工程物理所________組碩士班研究生入學考試

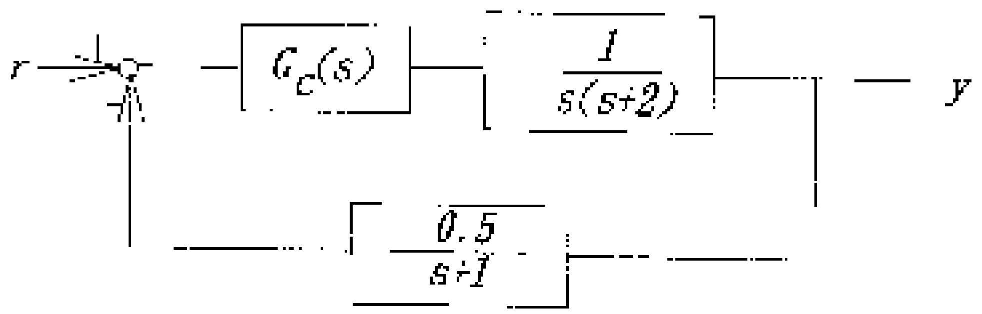
科目 控制系統　　科號 3307　共 3 頁第 1 頁　*請在試卷【答案卷】內作答

1. Consider the unity-feedback system with transfer function

$$G(s)=\frac{1}{s(s-1)(s^2+2s+2)}$$

Design a controller, as simple as possible, to stabilize the system.(20%)

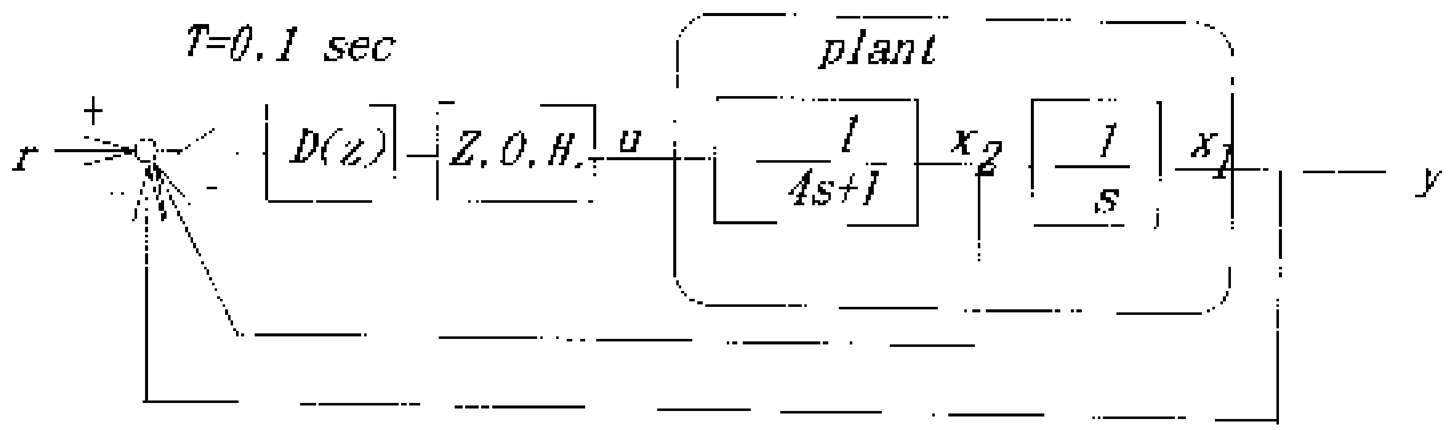
2. Consider the system shown below.



a) If $G_c(s)=k$ and $r(t)=1$, for $t\geq 0$. What is the steady-state error of the closed-loop system. Explain why the steady-state error exists.(10%)

b) Is it possible to design $G_c(s)$ so that the position error is zero? You must give the reason. If not, what would you do to achieve zero position error. (7%)

3. Consider the system shown below.



$$D(z)=\frac{4z-2.}{z-0.7}$$

a) Derive the discrete state model of the plant according to the state variables defined in the figure.(10%)

b) Derive the state model of the closed-loop system.(10%)

4. Consider a linear system described by the differential equation

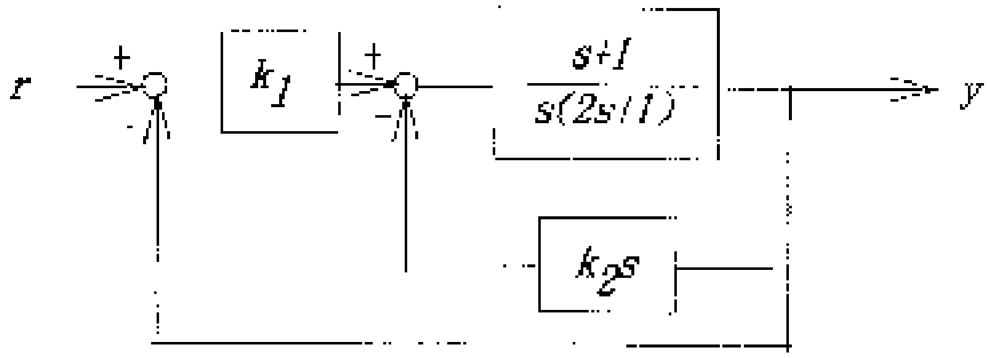
$$\ddot{y}+2\dot{y}+y=\dot{u}+u$$

Choose $x_1=y$ and $x_2=\dot{y}-u$

a) Derive the state model.(5%)

b) Which mode is controllable?(8%)

5. Consider the system shown below. Find the ranges of k so that the system are stable and have velocity error smaller than 10%. (Indicate the allowable region on the k_1-k_2 plane). (10%)



6. Pontryagin's Minimum Principle:
A control $u^* \in U$, which causes the system

$$\dot{x}(t)=f(x(t),u(t),t)$$

to follow an admissible trajectory that minimizes the performance measure

$$J(u)=h(x(t_f),t_f)+\int_{t_o}^{t_f} g(x(t),u(t),t)dt,$$

is sought. In terms of Hamiltonian

$$\mathrm{H}(x(t),u(t),p(t),t)=g(x(t),u(t),t)+p^T(t)[f(x(t),u(t),t)],$$

necessary conditions for u^* to be an optimal control are

$$\dot{x}^*(t)=\frac{\partial \mathrm{H}}{\partial p}(x^*(t),u^*(t),p^*(t),t),$$

$$\dot{p}^*(t)=-\frac{\partial \mathrm{H}}{\partial x}(x^*(t),u^*(t),p^*(t),t),$$

$$\min \mathrm{H}(x^*(t),u(t),p^*(t),t)=\mathrm{H}(x^*(t),u^*(t),p^*(t),t)$$

or equivalently

$$\mathrm{H}(x^*(t),u^*(t),p^*(t),t)\le \mathrm{H}(x^*(t),u(t),p^*(t),t), \qquad (1)$$

for all $t\in[t_o,t_f]$

and

$$\left[\frac{\partial h}{\partial x}(x^*(t_f),t_f) - p^*(t_f)\right]^T \delta x_f +$$
$$\left[H(x^*(t_f),u^*(t_f),p^*(t_f),t_f) + \frac{\partial h}{\partial t}(x^*(t_f),t_f)\right]\delta t_f = 0 \qquad (2)$$

Remark: 1. Superscript * means optimal value,

2. $p(t)$ is the costate,

3. Equation (2) is the boundary condition.

Consider the system having the state equations

$$x_1(t) = x_2(t)$$

$$x_2(t) = -x_2(t) + u(t)$$

with initial conditions $x(t_o) = x_o$. The performance measure to be minimized is

$$J(u) = \int_{t_o}^{t_f} \frac{1}{2}\left[x_1^2(t) + u^2(t)\right]dt$$

t_f is specifed and the final state $x(t_f)$ is free.

a) Find necessary conditions for an unconstrained control to minimized J. (7%)

b) Find necessary conditions for optimal control if

$$-1 \le u(t) \le +1 \qquad \text{for all } t \in [t_o, t_f]$$

(13%)