國立清華大學命題紙

八十五學年度 經濟學 系（所）＿＿＿組碩士班研究生入學考試

科目 微積分與統計　科號 4703　共 二 頁第 一 頁　*請在試卷【答案卷】內作答

1. Give the equations $x^2 + y^2 + z^2 = 5$, $xyz = -2$, and suppose that x and y are differentiable functions of z. Find dx/dz. (15 points)

2. Let r_1 and r_2 be the roots of the equation $x^2 - kx + (k-1) = 0$, for $k \in R$. Find the value of k for which $r_1^2 + r_2^2$ is a minimum. (15 points)

3. By considering

$$\frac{d^n}{dy^n}\int_0^1 x^y dx,$$

prove that

$$\int_0^1 (\log x)^n dx = (-1)^n n!.$$ (10 points)

4. Let the production function for good 1 be $F(K_1, L_1)$ and that for good 2 be $G(K_2, L_2)$, where K_i and L_i $(i = 1, 2)$ denote the amount of capital and labor used in the production of good i. Define the production possibility set Z as

$$Z = \{(q_1, q_2) \mid 0 \le q_1 \le F(K_1, L_1), 0 \le q_2 \le G(K_2, L_2),$$
$$K_1 + K_2 \le \overline{K}, L_1 + L_2 \le \overline{L}, K_i \ge 0, L_i \ge 0, i = 1, 2,$$
$$(\overline{K}, \overline{L}) \in R^2_{++}\}.$$

Prove that Z is a convex set if both F and G are concave functions. (10 points)

(1) The joint probability function of X, Y, and Z is given below:

(x,	y,	z)	f(x, y, z)
0	0	0	0.125
0	0	1	0.125
0	1	0	0.100
1	0	0	0.080
0	1	1	0.150
1	0	1	0.120
1	1	0	0.090
1	1	1	0.210

(a) Write out the joint probability distribution of X and Y. (5 points)

(b) Write out the marginal probability distributions of X. (5 points)

(c) Write out the conditional probability distribution of X given that Z = 0. (5 points)

(d) Define V=XY. Write out the probability distribution of V and compute the mean of V. (5 points)

(2) Suppose two economists estimate μ (the average expenditure of Taipei families on food), with two different, unbiased, and statistically independent estimates $\bar{x}_1$ and $\bar{x}_2$. The standard deviation of $\bar{x}_2$ is four times as large as the standard deviation of $\bar{x}_1$. Consider the following four ways of combining $\bar{x}_1$ and $\bar{x}_2$ to get an overall estimate:

(1) $\hat{\mu}_1 = (1/2)[\bar{x}_1 + \bar{x}_2]$

(2) $\hat{\mu}_2 = (2/3)\bar{x}_1 + (1/3)\bar{x}_2$

(3) $\hat{\mu}_3 = (3/4)\bar{x}_1 + (1/4)\bar{x}_2$

(4) $\hat{\mu}_4 = \bar{x}_1$

(a) Which of the four do you prefer? Why? (5 points)

(b) Find an even more efficient unbiased estimator of μ. (5 points)

(3) Find the maximum likelihood estimator of θ for the density function of X: $f(x|\theta) = \theta e^{-\theta x}$, $x > 0$, based on a random sample of size n. (10 points)

(4) Under the classical linear regression assumptions, the least squares regression equation estimated from 52 observations is

$$Y_t = 12 + 0.8\,X_t + e_t \qquad R^2 = 0.6$$

Also for α (significance level) = 0.05, $F_{1,40} = 4.08$ and $F_{1,50} = 4.00$.

Please use the information above to carry out the test for the existence of a linear relation between X and Y. Please conduct both F test and t test. (10 points)