

I. 下列語句是恆真的 (tautological)、適真的 (contingent) 或矛盾的 (contradictory)?

1.  $((P \rightarrow (Q \vee R)) \wedge (Q \rightarrow \sim R)) \leftrightarrow (\sim(\sim P \rightarrow Q \wedge R) \vee \sim(Q \leftrightarrow R))$  (10%)

2.  $(\forall x)(\forall y)(\forall z)((Rxy \wedge Ryz) \rightarrow Rxz) \wedge (\forall x)(\exists y)Rxy \wedge (\forall x)(Rxx \rightarrow (\forall y)(Rxy \rightarrow \sim Ryx))$  (10%)

II.

1.  $\{(\exists x)(Ax \wedge Bx) \rightarrow (\forall x)(Cx \rightarrow Dx), (\exists x)(Cx \wedge Dx), (\forall x)(Ax \rightarrow \sim Bx)\}$  是否為一致的 (consistent)? (10%)

2. 下列論證是否為有效的?

$$(\exists x)(Ax \wedge Bx) \rightarrow (\forall x)(Cx \rightarrow Dx)$$

$$(\exists x)(Cx \wedge Dx)$$

$$\therefore (\forall x)(Ax \rightarrow \sim Bx)$$

(10%)

III. 若 ‘?’、‘#’、‘\*’、‘×’ 為語句邏輯的邏輯連詞 (connectives)，其意義由下列真值表決定 (T 表示真值、F 表示假值)：

$P$	$Q$	$P?Q$	$P\#Q$	$P*Q$	$\times P$
T	T	F	T	T	F
T	F	T	T	F	F
F	T	F	T	F	T
F	F	T	F	F	T

則請用 ‘#’、‘\*’、‘×’ 三個邏輯連詞表示  $P?Q$ 。

(10%)

IV. 請將下列英文句子翻譯成邏輯句式 ( $\forall x$ : “for all  $x$ ”,  $\exists x$ : “for some  $x$ ”):

1. Someone is loved by everyone. (domain: human beings;  $Lxy$ : “ $x$  loves  $y$ ”) (2 %)
2. No one who attends college is ignorant. ( $Px$ : “ $x$  is a person”;  $Ax$ : “ $x$  attends college”;  $Ix$ : “ $x$  is ignorant”) (3 %)
3. No one learns anything unless he teaches it to himself. ( $Px$ : “ $x$  is a person”;  $Lxy$ : “ $x$  learns  $y$ ”;  $Txyz$ : “ $x$  teaches  $y$  to  $z$ ”) (5 %)
4. Everyone loves exactly one person. (domain: human beings;  $Lxy$ : “ $x$  loves  $y$ ”;  $Sxy$ : “ $x$  is the same person as  $y$ ”) (5 %)
5. Everyone admires the most intelligent person in the world. (domain: human beings;  $Ixy$ : “ $x$  is more intelligent than  $y$ ”;  $Axy$ : “ $x$  admires  $y$ ”;  $Sxy$ : “ $x$  is the same person as  $y$ ”) (5 %)

V. 請使用一套你熟悉的推論規則證明下列各題:

1.  $P \rightarrow (Q \rightarrow R)$   
 $\sim S \rightarrow (P \vee R)$   
 $P \rightarrow Q$   
 $\therefore R \vee S$  (10 %)
2.  $\vdash (\forall y)(\exists x)(Ax \rightarrow By) \leftrightarrow (\forall y)((\forall x)Ax \rightarrow By)$  (10 %)
3.  $(\forall x)((Ax \vee Bx) \rightarrow (Cx \wedge Dx))$   
 $\therefore (\exists x)(Ax \vee Cx) \rightarrow (\exists x)Cx$  (10 %)