

1. (10%)

- (a) (2%) Let $X = 11101001$, where X is a number in 2's complement representation. What is X (in decimal)?
- (b) (2%) If 11101001 is a number in 2's complement representation and is equal to $4X$, what is X (in 8-bit 2's complement)?
- (c) (2%) Find the complement of the expression $(A' + B)(C D' + E')$.
- (d) (2%) If $a \text{ XOR } b = a \text{ OR } b$, then what value is $a \text{ AND } b$?
- (e) (2%) Is the following identity true, $(a \text{ NAND } b) \text{ NAND } c = a \text{ NAND } (b \text{ NAND } c)$? Prove your answer.

2. (10%)

Parenthesis "[]" and "(")" are often used as string delimiter. Write a procedure *check_p* to check if the use of parenthesis in a string is balanced. For example, the use of parenthesis in $\{(ab)cd\}$ is balanced but the use of parenthesis in $[(a + b) * c]$ is not.

3. (3%)

Which sorting method is the slowest one in worst-case behavior?

- (a) merge sort (b) heap sort (c) quick sort (d) radix sort.

4. (3%)

Which data structure is the best in representing the sparse matrices?

- (a) tree (b) link list (c) array (d) queue or stack.

5. (8%)

(a) (2%) We can apply two techniques to speed up union-find operations. Please briefly describe these two techniques.

(b) (6%) Let A, B, C, D, E and F be six independent sets initially. Apply the above two speed-up techniques to perform the following union-find operations to the given sets,

UNION(A, B), UNION(B, C), UNION(D, F), UNION(D, E), UNION(A, D),
FIND(B), FIND(F).

Give the resultant tree representations.

6. (11%)

(a)(6%) How many binary trees can be determined by the inorder: HDIBEAFCG and the preorder: ABDHIECFG. Please draw all of them.

(b)(5%) Please draw the threaded binary tree corresponding to the tree in (a).

7. (10%)

How many bit strings of length 10 over the alphabet $\{a, b, c\}$ have either exactly three a 's or exactly four b 's?

8. (8%)

Let m be a positive integer greater than 1. Show that the relation

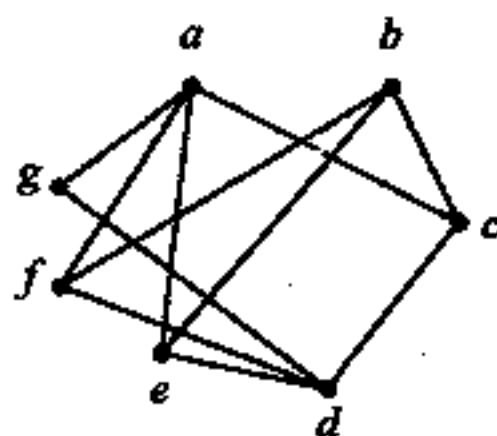
$$R = \{(a, b) \mid a \equiv b \pmod{m}\}$$

is an equivalence relation on the set of integers.

9. (7%)

A graph $G = (V, E)$ is called a bipartite graph if the set of vertices V can be partitioned into two nonempty subsets X and Y such that there is no edge in E joining two vertices in X or two vertices in Y .

Is the graphs G_1 bipartite? Justify your answer briefly.



G_1

10. (5%)

Construct Huffman binary codes for eight messages whose probabilities of appearance are 0.2, 0.19, 0.18, 0.14, 0.11, 0.08, 0.06, and 0.04.

11. (6%)

Show that among six persons, either there are three persons who are mutual friends or there are three persons who are complete strangers to each other.

12. (6%)

Suppose we toss a coin r times. There are 2^r possible sequences of outcomes. Let a_r denote the number of sequences that heads never appear on successive tosses. Find the recurrence relation of a_r .

13. (6%)

Show that the language

$$L = \{a^k \mid k = i^2, i > 0\}$$

is not a finite state language.

14. (7%)

Show that the graph in the following figure has no Hamiltonian path.

