

- (10%) Prove by induction that  $(11)^{n+1} + (12)^{2n-1}$  is divisible by 133 for any positive integer  $n$ .
- (10%) Determine the number of integers between 1 and 100 that are divisible by any of the integers 2, 3, and 5.
- (10%) Construct a grammar for the language  $L = \{a^i b^{2i} \mid i \geq 1\}$ .
- (10%) Determine the number of ways to seat 5 boys in a row of 11 chairs, such that no two boys are next to each other.
- (10%) Given a positive integer  $t$ , determine the number of integer solution sets to the following equations:

$$\begin{cases} x + y + z = 2t + 1 \\ x + y \geq z \\ y + z \geq x \\ x + z \geq y \\ x, y, z \geq 0. \end{cases}$$

- (10%) Let  $R$  be a binary relation on  $A$ . Give the definitions for the following terms: (a) reflexive relation, (b) symmetric relation, (c) antisymmetric relation, (d) transitive relation, and (e) transitive extension of  $R$ .
- (10%)
  - What is the pigeonhole principle?
  - A chess player wants to prepare for a championship match by playing some practice games in 77 days. She wants to play at least one game a day but no more than 132 games altogether. Use the pigeonhole principle to prove that no matter how she schedules the games, there is a period of consecutive days within which she plays exactly 21 games.
- (10%) Prove that an undirected graph possesses an eulerian path if and only if it is connected and has either zero or two vertices of odd degree.

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離散結構

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頁第 2 頁

請在試卷【答案卷】內作答

9. (10%) Use the method of generating functions to find the convolution  $a * b$ , where  $a_r = 3^r$  and  $b_r = 2^r$ ,  $r \geq 0$ .

10. (10%) The Fibonacci numbers are defined by the following recurrence:

$$\begin{cases} F_0 = 0, \\ F_1 = 1, \\ F_i = F_{i-1} + F_{i-2}, \text{ for } i \geq 2. \end{cases}$$

Derive an explicit expression for  $F_i$ .