

國 立 清 華 大 學 命 題 紙

九十三學年度 資訊系統與應用研究 系(所) 甲 組碩士班研究生招生考試

科目 離散結構 科號 3102 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1. (10%) Suppose that R_1 and R_2 are equivalence relations on the set S . Determine whether each of these combinations of R_1 and R_2 must be an equivalence relation.
 (a) $R_1 \cup R_2$ (b) $R_1 \cap R_2$ (c) $R_1 \oplus R_2$
2. (10%) Let $p(n)$ denote the number of different equivalence relations on a set with n elements. Show that $p(n)$ satisfies the recurrence relation $p(n) = \sum_{j=0}^{n-1} C(n-1, j)p(n-j-1)$ and the initial condition $p(0) = 1$.
3. (10%) Answer questions for the poset $(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, /)$, where the relation m / n if m divides n evenly.
 (a) Find the maximal elements.
 (b) Is there a greatest element?
 (c) Find all upper bound of $\{2, 9\}$.
 (d) Find the least upper bound of $\{2, 9\}$, if it exists.
 (e) Find the greatest lower bound of $\{60, 72\}$, if it exists.
4. (10%) The eccentricity of a vertex in an unrooted tree is the length of the longest simple path beginning at this vertex. A vertex is called a center if no vertex in the tree has smaller eccentricity than this vertex. Show that a tree has either one center or two centers that are adjacent.
5. (10%) Show that if T is a binary tree of height h and order p (that is, the number of vertices of T). Then, $h+1 \leq p \leq 2^{h+1} - 1$.
6. (10%) Suppose in an island, all people are either a knight or a knave, a knight always tells the truth and a knave always lies. If you encounter two persons A and B, what are the types of persons A and B if A says "B is a knight" and B says "The two of us are opposite types"?
7. (10%) Show that $2^n - 1$ is prime then n is prime.
8. (10%) Prove or disprove $\left\lfloor \frac{x}{2} \right\rfloor = \left\lfloor \frac{x+1}{2} \right\rfloor$ for all real number x .
9. (10%) Find the coefficients of x^{10} in the power series of the function

$$(1 + x^5 + x^{10} + x^{15} + \dots)^3$$
10. (10%) Solve the simultaneous recurrent relations

$$a_n = 3a_{n-1} + 2b_n$$

$$b_n = a_{n-1} + 2b_n \text{ with } a_0=1 \text{ and } b_0=2.$$