國　立　清　華　大　學　命　題　紙

八十八學年度 電子工程 系（所）＿＿＿組碩士班研究生招生考試

科目 工程數學 科號 4701 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (A) Find the general solution for $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=e^{-x}\cos(2x+1)$。 (5%)

   (B) $f(x)=\begin{cases}1 \text{ for } 0\le x<1\\ 0 \text{ for } 1\le x<2\end{cases}$ and $f(x+2)=f(x)$，find the Laplace transform of $f(x)$ and solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=f(x)$, $y(0)=0, y'(0)=0$。 (10%)

2. (A) From the properties of Dirac delta function, expand $\delta(1-4t^2)$ as the sum of delta functions with simple argument; that is, find the parameters A_n and a_n such that $\delta(1-4t^2)=A_1\delta(t-a_1)+A_2\delta(t-a_2)+\dots$ holds. (5%)

   (B) Solve the following equation by Laplace transform: $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+2y=\delta(1-4t^2)$, $y(0)=0, y'(0)=0$。 (5%)

   (C) Is the solution $y(t)$ in (B) continuous at $t=\frac{1}{2}$? If not, how are $y(t^+)$ and $y(t^-)$ related? Explain how you can figure out this relationship simply from the equation itself without actually solving for the solution. (5%)

3. Let P_3 be the set of all polynomials with degree less than 3.

   (a) (5 %) Find the transition matrix S from the ordered basis $[1, x, x^2]$ to the ordered basis $[1, 2x, 4x^2-2]$.

   (b) (5 %) Let D be the differentiation operator on P_3. Find the matrix A representing D with respect to the basis $[1, 2x, 4x^2-2]$.

4. Show the following Fourier transform theorems: (12%)

   (a) convolution theorem $F\{f*g\}=\sqrt{2\pi}F\{f\}F\{g\}$.

   (b) shifting theorem : $F\{f(x-a)\}=e^{-j\omega a}F\{f(x)\}$.

   (c) autocorrelation theorem : $F\{\int f(\xi)f(\xi-x)d\xi\}=\sqrt{2\pi}|F\{f\}|^2$.

5. Evaluate the following integral: (10%)

$$\int_0^\infty \frac{x-\sin x}{x^3(x^2+a^2)}dx,\ a>0.$$

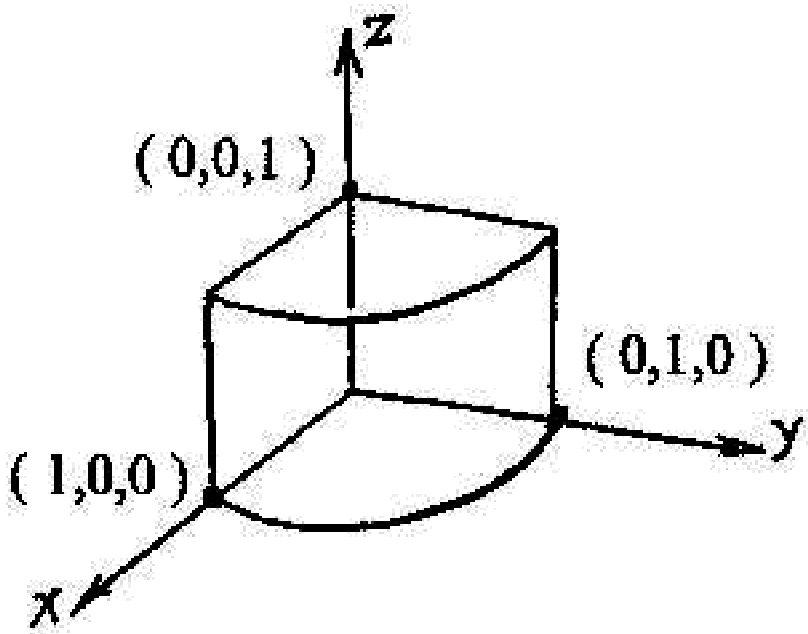
6. Show that if a > 0,

$$\int_0^\infty \frac{x-\sin x}{x^3(a^2+x^2)}dx = \frac{\pi}{2a^4}\left(\frac{a^2}{2}-a+1-e^{-a}\right). \quad (10\%)$$

7. (a) (4%) Find a vector field $\vec{F}(x,y,z)$ such that $\nabla\cdot\vec{F} = x^3 + y$.

(b) (11%) Compute the integral $\int_V (x^3+y)dxdydz$

using the divergence theorem, where V represents a quarter of the cylinder as shown below.



8. Solve the partial differential equation

$$\frac{\partial^3 u}{\partial t^3} = \frac{\partial u}{\partial x}$$

where u is a function of t and x satisfying $u(t, x=-\infty)=0$, $u(t=0,x)=0$,

$\frac{\partial u}{\partial t}\Big|_{t=0}=0$ and $\frac{\partial^2 u}{\partial t^2}\Big|_{t=0}=e^{8x}$. (13%)