八十八學年度 電子工程 系(所) ______ 組碩士班研究生招生考試
科目 電磁學 科號 4703 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. Prove that the following line integral from points A to B is independent of the path in the three dimensional space. (10%)

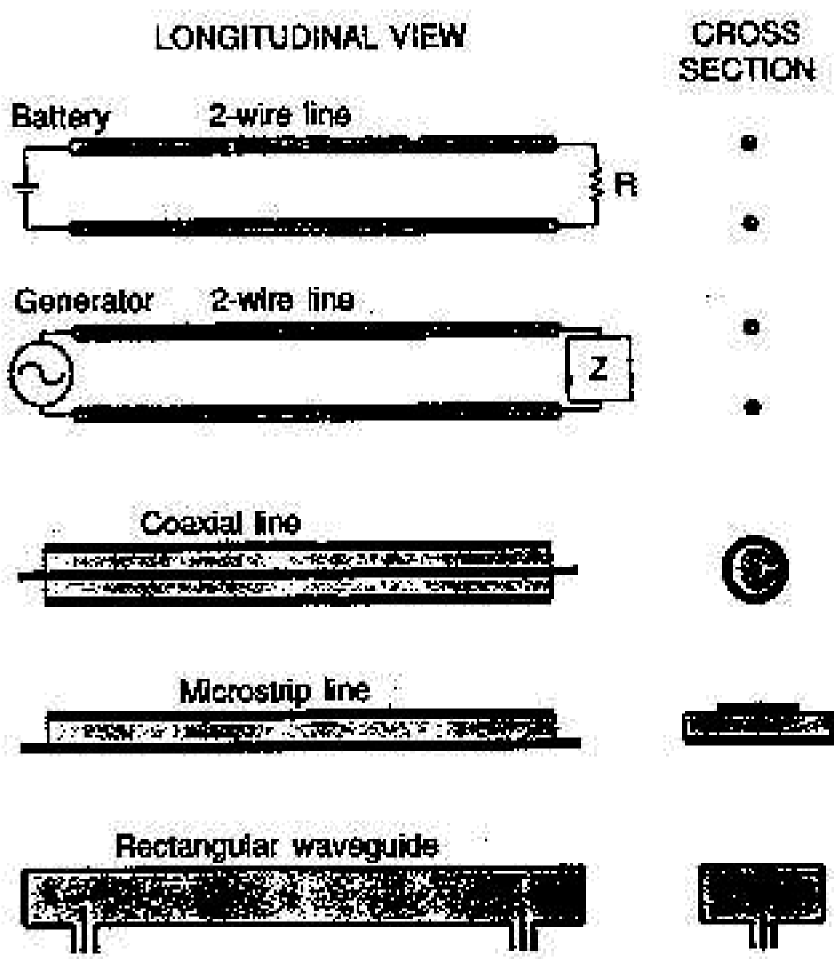
$$\int_A^B \nabla f(\mathbf{r}) \cdot d\mathbf{l}.$$

2. In the general orthogonal coordinates, prove that the divergence of a vector function $\mathbf{F}$ is given by

$$\nabla \cdot \mathbf{F} = \frac{1}{h_1 h_2 h_3} \left\{ \frac{\partial F_1 h_2 h_3}{\partial u_1} + \frac{\partial F_2 h_1 h_3}{\partial u_2} + \frac{\partial F_3 h_1 h_2}{\partial u_3} \right\}.$$

where F_i, h_i, and u_i $(i = 1, 2, 3)$ are the components, metric coefficients, and coordinates in the general orthogonal coordinate system, respectively. (15%)

3. The range of the operation frequencies of transmission lines are quite different. The longitudinal and cross sectional views of five transmission lines are given below. Explain the very factors causing their operation frequency to be limited. (10%)



4. A point charge q is placed in a linear, isotropic, and homogeneous dielectric medium of infinite extent. Find (a) the electric field intensity $\mathbf{E}$, (b) the electric flux density $\mathbf{D}$, (c) the polarization vector $\mathbf{P}$, and (d) the polarization volume charge density ρ_{pv}. (10%)

5. A uniform plane wave traveling in air with its electric field intensity given by

$$\mathbf{E}_i(y,t) = \hat{z}E_0\cos(\omega t - \beta y)$$

is normally incident on a perfect conductor boundary located at $y = 0$. If the measured distance between any two successive zero of the total electric field in air is 6 cm and the maximum value of the electric field intensity measured at $y = -75$ cm is 3 V/m, determine the following: (15%)
(a) The frequency (in GHz).
(b) The power density (in μW/cm^2).
(c) The instantaneous expression of the total electric field intensity $\mathbf{E}_i(y,t)$ in air.
(d) The instantaneous expression of the total magnetic field intensity $\mathbf{H}_i(y,t)$ in air.
(e) The maximum value of the total magnetic field intensity measured at $y = -75$ cm.

6. Show that the electric field intensity can be expressed in terms of the magnetic vector potential as

$$\mathbf{E} = -j\omega\left(\mathbf{A} + \frac{1}{k^2}\nabla\nabla\cdot\mathbf{A}\right),$$

where $k = \omega\sqrt{\mu\epsilon}$ is the phase constant in the unbounded medium. (10%)

7. It is known that for a thin, linear, center-fed dipole antenna of length $2h$, the induced current distribution is a standing wave. And, the pattern function for this antenna is

$$F(\theta) = \frac{\cos(k_0 h\cos\theta) - \cos(k_0 h)}{\sin\theta}.$$

This pattern function depends strongly on the length h/λ, where $\lambda = 2\pi/k_0$. Show that for a short antenna with $h/\lambda \ll 1$, the pattern function can be simplified as

$$F(\theta) = a\sin\theta$$

and find the constant a. (10%)

8. The electric field distribution of the TE_{10} mode in an air-filled rectangular waveguide of size $a \times b$ $(a > b)$ is known as:

$$\mathbf{E} = \hat{y} \sin\left(\frac{\pi}{a} x\right) e^{-j\beta z}.$$

This field can be decomposed as

$$\mathbf{E} = \hat{y} \frac{j}{2} \left\{ e^{-j\frac{\pi}{a}x} e^{-j\beta z} - e^{j\frac{\pi}{a}x} e^{-j\beta z} \right\}.$$

Thus, each of the two components of the field does not propagate along the axial z direction, but along a direction with an angle $\pm\theta$ from the z axis. Determine the angle θ and use it to express λ_g, the guided wavelength of TE_{10} mode, in terms of λ_0, the wavelength of a plane wave propagating in free space at the same frequency. (10%)

9. When a transmission line of characteristic impedance Z_0 and length ℓ connects a load to a generator of voltage V_g and internal impedance Z_g, the expressions of the voltage $V(z)$ along the line is as complicated as

$$V(z) = V_g \frac{Z_0}{Z_0 + Z_g} \frac{1}{1 - \Gamma\Gamma_g e^{-j2\beta\ell}} \left(e^{-j\beta z} + \Gamma e^{-j2\beta\ell} e^{j\beta z}\right).$$

However, the expressions of the voltage $V(z)$ and the current $I(z)$ are quite similar. Write down the corresponding expression for the current $I(z)$. (10%)