

國 立 清 華 大 學 命 題 紙

九十三學年度 電子工程研究所 系(所) _____ 組碩士班入學考試

科目 工程數學 科號 2901 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Solve the equation $x^2 y'' - (x + x^2)y' + y = 0$ ($0 < x < \infty$). (18%)

2. Solve $f(t) = 2t^2 + \int_0^t \sin(4\tau)f(t-\tau)d\tau$. (7%)

3. For a partial differential equation: $\frac{\partial U}{\partial t} = \alpha^2 \frac{\partial^2 U}{\partial x^2}$, for $-\infty < x < \infty$ and $0 < t < \infty$. The initial condition is $U(x,0) = \cos(x)$. Please find the solution of $U(x, t)$ and $U(x, \infty)$. (15%)

4. For a partial differential equation:

$$\nabla^2 U - v^2 \frac{\partial^2 U}{\partial t^2} = 0$$

(a) Please find general solutions of $U(R;t)$ for this partial differential equation. (5%)

(b) In many cases, such solutions are in the time-harmonic form, that is

$$U(R;t) = U(x,y,z)e^{i\omega t}$$

If $\frac{\partial^2 U}{\partial x^2} = \frac{\partial^2 U}{\partial y^2} = 0$, please find a general solution of $U(z;t)$. (5%)

(c) Following (b), if at $z = \left(\frac{\omega}{2\pi v}\right)^{-1}$, the boundary condition is $U(z;t)=0$, please find the solution of $U(z;t)$. (5%)

5. Find the values of the integrals along the path C that is a counter-clockwise circle of radius 2:

(a) $\int_C \frac{s}{(s-1)(s-3)} ds$. (5%)

(b) $\int_C \frac{s}{\sin(s)} ds$. (5%)

6. Evaluate the integral $\int_{-\infty}^{+\infty} \frac{e^{az}}{(z+1)(z+4)} dz$ by appropriate contour integration, assuming that a is real. Show a diagram of the contour that you use. (20%)

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7. (a) Find the Fourier series of the function as shown in Figure 1. (8%)

$$f(t) = \begin{cases} 0 & \text{when } -2 < t < -1 \\ k & \text{when } -1 < t < 1 \\ 0 & \text{when } +1 < t < 2 \end{cases} \quad T = 4$$

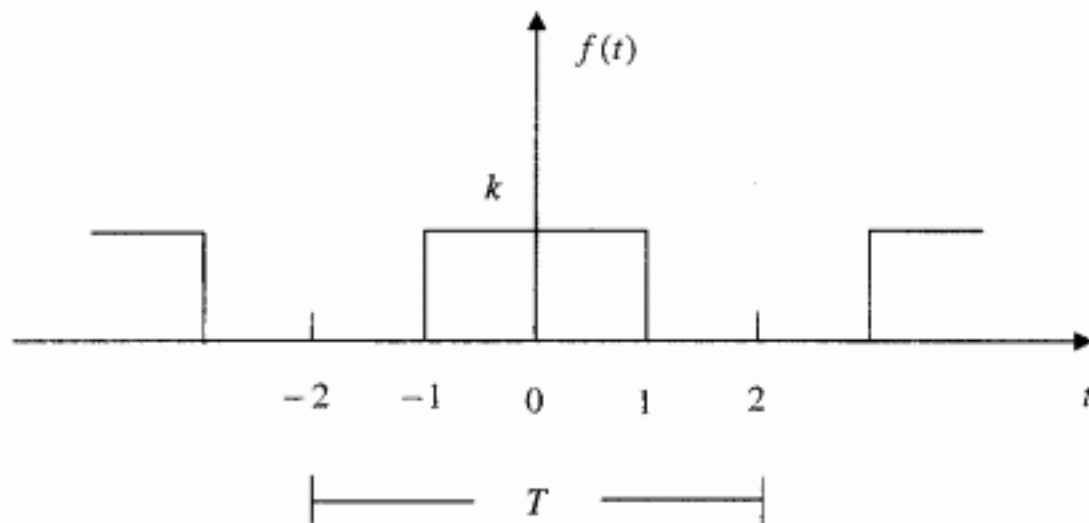


Figure 1

(b) As T approaches infinity in Fig. 1 (single pulse), find the Fourier integral representation of the function. (7%)