國 立 清 華 大 學 命 題 紙

八十八學年度 通訊工程研究所系（所） 乙 組碩士班研究生招生考試

科目 基礎數學 科號 4301 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (10%) Find the coefficient of the term x^{23} in $(1+2x^5+3x^9)^{100}$.

2. (10%) Let $A(x)$ be the ordinary generating function of the sequence $\{a_i, i=0,1,2,\ldots\}$. Find the ordinary generating function of the sequence $\{q_i, i=0,1,2,\ldots\}$, where $q_n = \sum_{i=n+1}^{\infty} a_i$ (Assume that all the q's are finite.)

3. Assume that the sequence $\{p_i, i=0,1,2\ldots\}$ satisfies the following recurrence relation

$$\begin{aligned}(1+a)p_i &= ap_{i-1}+p_{i+1}, \quad i=1,2,\ldots \\ ap_0 &= p_1.\end{aligned}$$

   (a) (10%) Assuming that $p_0 = 1$, find p_i, $i=1,2,\ldots$.

   (b) (10%) Assuming that $\sum_{j=0}^{\infty} p_j = 1$, find p_i, $i=0,1,2,\ldots$.

4. Let $\{X_i, i=1,2,\ldots\}$ be a sequence of independent exponential random variables with rate α. That is, the probability density function of X_i is

$$f_{X_i}(t) = \alpha e^{-\alpha t}, \quad t \geq 0.$$

   Let Y be an exponential random variable with rate β and assume that Y is independent of $\{X_i, i=1,2,\ldots\}$. Find the following:

   (a) (7%) $\Pr(X_1 < Y)$

   (b) (7%) $\Pr(X_1 < Y, X_1 + X_2 > Y)$

   (c) (7%) $\Pr(\sum_{i=1}^{n} X_i < Y, \sum_{i=1}^{n+1} X_i > Y)$

   (d) (7%) the density function of $\min(X, Y)$.

5. Consider a casher of a supermarket who works for a random length of time T everyday. During his/her work time, customers arrive and each customer pays an independent, exponentially distributed (with mean m) amount of money. Further more, assume that the number of customers arrived during an interval of length τ is an independent Poisson random variable, i.e.

$$\Pr(N(\tau) = i) = \frac{e^{-\lambda\tau}(\lambda\tau)^i}{i!}.$$

(a) Find the <u>mean</u> and the <u>characteristic function</u> of the amount of money received by the casher, assuming that

  i. (10%) T is uniformly distributed between a and b, where $0 < a < b$;

  ii. (10%) T is exponentially distributed with mean $\bar{T}$.

(b) (12%) In case ii, what is the distribution function of the amount of money received by the casher during his/her work time?