

國 立 清 華 大 學 命 題 紙

97 學年度工程與系統科學系乙、丙組、先進光源科技學位學程乙組及核子工程與科學研究所甲組
碩士班入學考試

科目 工程數學 科目代碼 2901、3001、3201、3101 共 2 頁第 1 頁 *請在【答案卷卡】內作答

1. (a) Evaluate

$$\int_{(0,0)}^{(2,1)} (5y^3 + 20x^4y^2)dx + (15xy^2 + 8x^5y - 3)dy$$

along the path $x^4 - 6xy^3 = 4y^2$.

(5%)

(b) Let $\vec{v} = rz^2\hat{e}_z$, which is given in cylindrical coordinates

(r, θ, z) . Evaluate $\oint_S \hat{n} \cdot \vec{v} dA$ where S is the surface of

the cone V (see figure).

(5%)

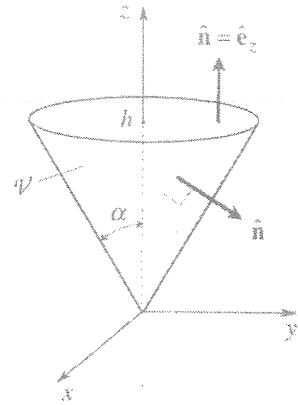


Figure The cone V

2. (a) Find an orthonormal set of the linear independent set $\{(2,0,0), (1,1,0), (3,3,3)\}$ using Gram-Schmidt orthogonalization process. (5%)

(b) Matrix $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 4 & 3 \\ -1 & 3 & 4 \end{pmatrix}$ can be expressed as $Q^T D Q$ where D is a diagonal matrix and Q is an

orthogonal matrix. Find D and Q . (5%)

3. Find the *harmonic* function $u(x, y)$ in the semi-infinite strip $0 < x < \pi, y > 0$ such that

$$u(0, y) = u(\pi, y) = 0 \quad (y > 0),$$

$$u(x, 0) = 1 \quad (0 < x < \pi),$$

and $|u(x, y)| < M$, where M is some constant.

(10%)

4. Determine the residue of each of the following functions at each singularity:

(a) $\tan z$,

(b) $\frac{\sin z - z}{z^6}$,

(c) $z e^{1/z}$.

(10%)

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5. Solve by Fourier transform

$$u'''' + ku = w(x),$$

where k is constant and $w(x)$ can be expanded in a Fourier integral, and $u(x), u'(x), u''(x), u'''(x) \rightarrow 0$,
as $x \rightarrow \pm\infty$.

Hint: the Fourier transform of $f(x) = e^{-a|x|/\sqrt{2}} \sin\left(\frac{a}{\sqrt{2}}|x| + \frac{\pi}{4}\right)$ ($a > 0$) is $\frac{2a^3}{\omega^4 + a^4}$.

(10%)

6. Find the weighting function of the following equation to become a SLP (Sturm-Liouville Problem)
-type equation, $(1-x^2)y'' - xy' + \lambda y = 0$.

(5%)

7. Find the general solution $y(x)$ of the following differential equation

$$x^2 y'' - 2xy' + 2y = x \ln x.$$

(Hint: let $x = e^t$.)

(15%)

8. (a) Prove the following relations between Laplace transforms

$$L\{y'(t)\} = sL\{y(t)\} - y(0),$$

$$L\{t y(t)\} = -\frac{d}{ds} L\{y(t)\}.$$

(b) Solve the following problem using Laplace transform

$$t y'' + 2t y' + 2y = 0; \quad y(0) = 0.$$

(15%)

9. Prove the recurrence relations satisfied by Legendre polynomials

$$(k+1)P_{k+1}(x) - (2k+1)xP_k(x) + kP_{k-1}(x) = 0. \quad k = 1, 2, 3, \dots$$

(Hint : A generating function of the Legendre polynomials is

$$(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n.$$

Differentiate the above equation once with respect to t .)

(15%)