

科目：基礎數學 科號：3901 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

- (15%) Suppose that no three diagonals of a convex decagon (eight sides) meet at the same point inside the decagon.
 - What is the total number diagonals?
 - What is the total number of intersections between the diagonals?
 - Into how many line segments are the diagonals divided by their intersections?
- (5%) Five distinct letters are to be transmitted through a communication channel. A total of 15 blanks are to be inserted between the letters with at least three blanks between every two letters. In how many ways can the letters and blanks be arranged?
- (5%) Solve the difference equation $a_n + 2a_{n-1} = n + 3$ with the boundary condition $a_0 = 3$.
- (5%) Find the number of integers between 1 and 250 that are not divisible by the integers 2, 3, 5, and 7.
- (5%) Find the number of distinct bracelets of five beads made up of yellow, blue, and white beads. Two bracelets are said to be indistinguishable if the rotation of one will yields another. However, to simplify the problem, we assume that the bracelets cannot be flipped over.
- (5%) Eight people are planning vacation trips. There are three cities they can visit. Three of these eight people are in one family, and two of them are in another family. If the people in the same family must go together, find the number of ways the eight people can plan their trips.
- (5%) Eleven students plan to have dinner together for several days. They will be seated at a round table, and the plan calls for each student to have different neighbors at every dinner. For how many days can this be done?
- (5%) How long is a longest circular sequence of 1's and 0's such that no subsequence of r bits appears more than once in the sequence?
- (10%) There are two boxes of light bulbs. Box A contains 7 light bulbs rated at 75W and 3 bulbs rated at 50W. Box B contains 2 75W bulbs and 8 50W bulbs. The boxes are chosen at random with probabilities $\Pr(\text{box } A) = 2/3$ and $\Pr(\text{box } B) = 1/3$ because of their respective location. A light bulb is then selected at random from the chosen box. Given that a 50W light bulb is selected, what is the probability that it is selected from box A?

科目：基礎數學 科號：3901 共 2 頁第 2 頁 *請在試卷【答案卷】內作答

10. (10%) Each of three persons fires one shot at a target. Let A_i denote the event that the target is hit by person i , $i = 1, 2, 3$. If we assume that A_1, A_2, A_3 are mutually independent and if $\Pr(A_1) = 0.7$, $\Pr(A_2) = 0.9$, $\Pr(A_3) = 0.8$, compute the probability that exactly two people hit the target (i.e. one person misses).
11. (10%) Suppose that a man leaves for work between 8:00AM and 8:30AM and takes between 40 and 50 minutes to get to the office. Let X denote the time of departure and let Y denote the time of travel. If we assume that these two random variables are independent and uniformly distributed, find the probability that he arrives at the office before 9:00AM.
12. (10%) Let X_1, X_2 and X_3 be mutually independent random variables with exponential distributions with means 1, 2, 3, respectively. (That is, the p.d.f. of X_i is

$$\frac{e^{-x/i}}{i}$$

for $i = 1, 2, 3$). Find the moment-generating function of the sum $X_1 + 2X_2 + 3X_3$.

13. (10%) Assume that objects locate randomly on the plane such that for any region of area A the number of objects in that region has a Poisson distribution with mean λA . Consider an arbitrary point in the plane and let X denote its distance from its nearest object (where distance is measured in the usual Euclidean manner). Find $\Pr(X > t)$.