

國立清華大學命題紙

九十二學年度 通訊工程研究所 (所) 乙 組碩士班研究生招生考試

科目 基礎計算機科學 科號 3101 共 3 頁第 1 頁 \*請在試卷【答案卷】內作答

1. (5 points) The president of NTHU promises to give the Institute of Communications Engineering one dollar in first day, two dollars in 2nd day, four dollars in 3rd day, eight dollars in 4th day, sixteen dollars in 5th day, and so on. How many dollars will the Institute of Communications Engineering receive in the end of 64th day? Solve this problem by using recurrence relation.

2. (6 points) Let  $a_r$  denote the number of ways to put  $r$  same balls into five different boxes with the constraints that the number of balls in the first two boxes are an even number less than or equal to ten, and the number of balls in other three boxes are between three and five inclusively. Determine the generating function of the numeric function  $a$ .

3. (6 points) Let  $\{B, C, S\}$  be the set of nonterminals with  $S$  being the starting symbol. Let  $\{a, b, c\}$  be the set of terminals. Consider the following grammar:

$$S \rightarrow aSBC \mid aBC$$

$$CB \rightarrow BC$$

$$aB \rightarrow ab$$

$$bB \rightarrow bb$$

$$bC \rightarrow bc$$

$$cC \rightarrow cc$$

What strings are produced by this grammar  $G$ ?

4. (4 points) Which one of the following statement is false?

(a) In  $R^3$ , the vectors  $(1, 3, 2)^T$ ,  $(-3, 5, 4)^T$  and  $(0, 7, 5)^T$  are linearly independent.

(b) Suppose that  $\{x_1, \dots, x_n\}$  are linearly independent members of a vector space  $V$  and that  $\{y_1, \dots, y_m\}$  are vectors spanning  $V$ . Then  $n \leq m$ .

(c) If  $W_1$  and  $W_2$  are subspaces of a vector space  $V$ , then  $W_1 \cup W_2$  is also a subspace of  $V$ .

(d) If  $W_1$  and  $W_2$  are subspaces of a vector space  $V$ , then  $W_1 \cap W_2$  is also a subspace of  $V$ .

(e) If  $W_1$  and  $W_2$  are subspaces of a vector space  $V$ , then  $W_1 + W_2 = \{x + y : x \in W_1, y \in W_2\}$  is also a subspace of  $V$ .

5. (4 points) Which one of the following functions is not linear?

(a)  $f(x) = 3x - 2$  from  $R^1$  to  $R^1$ .

(b)  $f \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ y \\ x - 2y \end{pmatrix}$  from  $Q^2$  to  $Q^3$ .

(c)  $f \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7x + 4y - 4z \\ 4x - 8y - z \\ -4x - y - 8z \end{pmatrix}$  from  $R^3$  to  $R^3$ .

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(d)  $f(a + bi) = a - bi$  from  $C$  to  $C$  regarded as a 2-dimensional vector space over  $R$ .  $i$  is the unit imaginary number.

(e) none of the above.

6. (5 points) Solve

$$x + y + 2z = 1$$

$$2x - y + 3z = 2$$

$$3x + 5y = 4$$

7. (4 points) How many terms are there in the expansion of  $(a + b + c + d)^5$ ?

8. (8%) On a tree, the number of subtrees of a node is called the degree of that node. A node of degree zero is called a leaf node. If a tree has  $n_i$  nodes of degree  $i$ ,  $i = 1, 2, \dots, m$ , how many leaf nodes does it have?

9. (4%) Find all binary trees whose nodes appear in exactly the same sequence in both preorder and inorder.

10. (4%) Find all binary trees whose nodes appear in exactly the same sequence in both preorder and postorder.

11. (10%) Using the modulo-division hashing and the linear probing, store the keys shown below in an array with 19 elements. Show the result of the hashed list. How many collisions occurred? What is the density of the list after all keys have been inserted?

39 29 33 20 28 37 48 31 36

12. (10%) Compare the differences between array and linked list structures on all aspects that you can think of.

13. (10%) Describe a method to implement a queue of finite size using an array such that no movement of data is required when an entry is removed from the front of the queue.

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14. (10%) A binary tree has ten nodes. The inorder and preorder traversal of the tree are shown below. Draw the tree.

Preorder: JCBADFEIGH

Inorder: ABCEDFJGHI

15. (10%) Find the minimum spanning tree of the following graph.

