國 立 清 華 大 學 命 題 紙

98 學年度 動力機械 系（所） 甲、丙、丁 組碩士班入學考試

科目 工程數學 科目代碼 1003 1103、1203 共 2 頁第 1 頁 *請在【答案卷卡】內作答

1. Solve the following ordinary differential equations

   (i) $y'' = x\,(y')^3$ (10%)

   (ii) $x^2 y'' + (x-1)(x y' - y) = x^2 e^{-x}$ (10%)

2. Find the inverse Laplace transform of

$$F(s) = \frac{e^{-s}}{s(s+1)(s+2)}$$ (10%)

3. Given the matrix

$$\mathbf{A} = \begin{bmatrix} \frac{31}{40} & \frac{9\sqrt{3}}{40} \\ \frac{9\sqrt{3}}{40} & \frac{13}{40} \end{bmatrix},$$

compute $\lim_{n\to\infty} \mathbf{A}^n$ (10%)

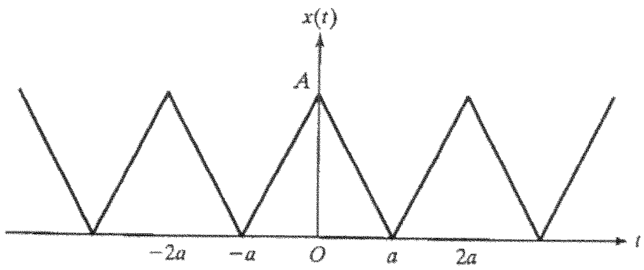
4. Evaluate the line integral of the normal derivative of a function $w(x,y)$ counterclockwise over the boundary curve C of the rectangle defined by $0 \le x \le 1$, and $0 \le y \le 2$, i.e. to evaluate the following integral

$$\oint_C \frac{\partial w}{\partial n}\, ds \quad \text{with} \quad w = e^x + e^{2y}$$ (10%)

5. Any periodic function $x(t)$, of period $2a$, can be expressed in the form of a complex Fourier series $x(t)=\sum_{n=-\infty}^{\infty} c_n e^{in\omega_o t}$ where ω_o is the fundamental frequency given as $\omega_o=\frac{\pi}{a}$. Using the relation $\int te^{kt}dt=\frac{e^{kt}}{k^2}(kt-1)$ to find the complex Fourier series expansion of the function below,

$$x(t)=\begin{cases} A(1+\frac{t}{a}), & -a\le t\le 0 \\ A(1-\frac{t}{a}), & 0\le t\le a \end{cases}$$

with the period $2a$ and the fundamental frequency ω_o . (15%)



6. The free vibration equation of a bar (length ℓ) along the axial direction can be expressed as

$$c^2\frac{\partial^2 u}{\partial x^2}(x,t)=\frac{\partial^2 u}{\partial t^2}(x,t).$$

The bar is fixed at $x=0$ and free at $x=\ell$ and the boundary conditions can be expressed as

$$u(0,t)=0,\quad t\ge 0$$

$$\frac{\partial u}{\partial x}(\ell,t)=0,\quad t\ge 0$$

The initial conditions can be stated as $u_o(x)$ and $\dot{u}_o(x)$. Using the method of separation of variables $U(x,t)=U(x)T(t)$ to find the eigenfunctions $u_n(x,t)$ and eigenvalues λ_n of this vibrating bar and the general free vibration solution. (15%)

7. Evaluate the following integrals by complex function theory

(i) $\int_0^\infty \frac{dx}{\sqrt{x}(x+4)(x-5)}$ (10%)

(ii) $\int_0^\infty \frac{\cos x}{x^{1-m}}dx$, $0<m<1$ (10%)