

國立清華大學命題紙

99 學年度 工業工程與工程管理學系工業工程組甲組 碩士班入學考試

科目 作業研究 科目代碼 1402 共 149 頁第 1 頁

\*請在【答案卷卡】作答

- 注意事項：(1) 不得使用計算器。  
 (2) 請依題號順序作答。  
 (3) 答案必須寫在答案卷上，並須依每一題規定的方式作答。  
 (4) 未依規定方式作答，酌量扣分。

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1. True (T) or False (F)? 以下是非題每一小題2分，但答錯一小題倒扣2分，最多只扣到本題0分，必須在答案卷畫出以下表格並在表格內填寫答案。

題目 1	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
答案										

(1)  $(x_1, x_2) = (2, 6)$  is an optimal solution of the following linear program

$$\begin{aligned} &\text{Maximize } 3x_1 + 5x_2 \\ &\text{subject to } x_1 \leq 4 \\ &\quad 2x_2 \leq 12 \\ &\quad 3x_1 + 2x_2 \leq 18, \quad x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

and  $(y_1, y_2, y_3) = (0, \frac{3}{2}, 1)$  is an optimal solution of its dual problem

$$\begin{aligned} &\text{Minimize } 4y_1 + 12y_2 + 18y_3 \\ &\text{subject to } y_1 + 3y_3 \geq 3 \\ &\quad 2y_2 + 2y_3 \geq 5, \quad y_1 \geq 0, y_2 \geq 0, y_3 \geq 0. \end{aligned}$$

We can conclude that  $(x_1, x_2, x_{new}) = (2, 6, 0)$  is an optimal solution of the following problem.

$$\begin{aligned} &\text{Maximize } 3x_1 + 5x_2 + 5x_{new} \\ &\text{subject to } x_1 + 3x_{new} \leq 4 \\ &\quad 2x_2 + 2x_{new} \leq 12 \\ &\quad 3x_1 + 2x_2 + x_{new} \leq 18, \quad x_1 \geq 0, x_2 \geq 0, x_{new} \geq 0. \end{aligned}$$

- (2) For any payoff table of a two-person zero-sum game, if mixed strategies are allowed, then there exists at least one stable solution.
- (3) We can construct a linear program which has exact two distinct feasible

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科目 作業研究 科目代碼 1402 共 14 頁第 2 頁

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solutions.

- (4) The feasible region of any given linear program must be a convex set.
- (5) The optimal solution of a linear program may not be an extreme point.
- (6) Once a primal LP has no feasible solution, we know that its dual is unbounded.
- (7) The simplex method can solve the maximum flow problem.
- (8) The following linear fractional program can be transformed into a linear program.

$$\begin{aligned} &\text{Maximize } f(\mathbf{x}) = \frac{\mathbf{c}\mathbf{x} + c_0}{\mathbf{d}\mathbf{x} + d_0} \\ &\text{subject to } \mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}. \end{aligned}$$

- (9) In order to obtain a basic feasible solution, the surplus variables must be 0 in the two-phase method.
- (10) In an LP model, adding an additional constraint can improve the value of the objective function.

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2. Consider the following linear program.

$$\begin{aligned} &\text{Maximize } 5x_1 + 2x_2 + 3x_3 \\ &\text{subject to } x_1 + 5x_2 + 2x_3 \leq b_1 \\ &\quad \quad \quad x_1 - 5x_2 - 6x_3 \leq b_2, x_1, x_2, x_3 \geq 0. \end{aligned}$$

Let  $x_4$ , and  $x_5$  be the slack variables of constraints 1 and 2 respectively. Specific constant values of  $b_1$  and  $b_2$  produce the following tableau.

Basic Variable	Coefficients of:						RHS
	$z$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$z$	1	0	A1	A2	A3	0	D
$x_1$	0	1	B1	2	B2	0	30
$x_5$	0	0	C1	-8	C2	1	10

Determine  $b_1, b_2, A1, A2, A3, B1, B2, C1, C2$ , and  $D$ . 必須在答案卷畫出以下表格並在表格內填寫答案。

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科目 作業研究 科目代碼 1402 共 10 頁第 3 頁

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題目 2	$b_1$	$b_2$	A1	A2	A3	B1	B2	C1	C2	D
答案										

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3. Consider the following linear program.

$$\begin{aligned} & \text{Maximize} && c_1x_1 + c_2x_2 \\ & \text{subject to} && a_{11}x_1 + a_{12}x_2 \leq b_1 \\ & && a_{21}x_1 + a_{22}x_2 \leq b_2 \\ & && a_{31}x_1 + a_{32}x_2 \leq b_3, x_1, x_2 \geq 0. \end{aligned}$$

Let  $x_3, x_4,$  and  $x_5$  be the slack variables of constraints 1, 2, and 3 respectively. The optimal tableau is given as follows.

Basic Variable	Coefficients of:						RHS
	$z$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$z$	1	9/2	0	0	0	5/2	40
$x_3$	0	1	0	1	0	0	4
$x_2$	0	3/2	1	0	0	1/2	9
$x_4$	0	-3	0	0	1	-10	6

Solve the following linear program with one additional constraint.

$$\begin{aligned} & \text{Maximize} && c_1x_1 + c_2x_2 \\ & \text{subject to} && a_{11}x_1 + a_{12}x_2 \leq b_1 \\ & && a_{21}x_1 + a_{22}x_2 \leq b_2 \\ & && a_{31}x_1 + a_{32}x_2 \leq b_3 \\ & && 2x_1 + 3x_2 \leq 24, x_1, x_2 \geq 0. \end{aligned}$$

Determine the optimal value and the optimal primal solution.

必須在答案卷畫出以下表格並在表格內填寫答案。

題目 3	optimal value	$x_1^*$	$x_2^*$	$x_3^*$	$x_4^*$	$x_5^*$
答案						

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4. Explain the following terminology in queuing system.:

- (1)  $M/G/s/K$  Model (8%)
- (2) Jackson Network (7%)

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5. Airplanes arrive for take-off at the runway of an airport according to a Poisson process at a mean rate of 20 per hour. The time required for an airplane to take off has an exponential distribution with a mean of 2 minutes, and this process must be completed before the next airplane can begin to take off. Because a brief thunderstorm has just begun, all airplanes which have not commenced take-off have just been grounded temporarily. However, airplanes continue to arrive at the runway during the thunderstorm to await its end. Assuming steady-state operation before the thunderstorm, determine the expected number of airplanes that will be waiting to take off at the end of the thunderstorm if it lasts 30 minutes.

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6. Customers arrive at a fast food restaurant with one server according to a Poisson process at a mean rate of 20 per hour. The server has just resigned, and the two candidates for the replacement are  $X$  (fast but expensive) and  $Y$  (slow but inexpensive). Both candidates would have an exponential distribution for service times with  $X$  having a mean of 1.2 minutes and  $Y$  having a mean of 1.5 minutes. Restaurant revenue per month is given by  $\$3,000/W$  where  $W$  is the expected waiting time (in minutes) of a customer in the system.

- (1) Determine the expected waiting time of a customer in the system by hiring either  $X$  or  $Y$ . (10%)
- (2) Determine the upper bound on the difference in their monthly compensations that would justify hiring  $X$  rather than  $Y$ . (10%)