

國立清華大學 100 學年度碩士班入學考試試題

系所班組別：工業工程與工程管理學系(丁組)

考試科目 (代碼)：微積分(一般生) (1901)

共 2 頁，第 1 頁 \*請在【答案卷】作答

1. Find the following limits respectively. If it does not exist, explain why.

(a)  $\lim_{x \rightarrow 1} \left( \frac{1}{\ln x} - \frac{1}{x-1} \right)$  (5 pts)

(b)  $\lim_{x \rightarrow 0} \frac{2x^2}{1 - \cos 3x}$  (5 pts)

(c)  $\lim_{x \rightarrow \infty} \left( \frac{2}{\pi} \tan^{-1} x \right)^x$  (5 pts)

2. Evaluate  $\int_0^1 x^{15} \sqrt{1+3x^8} dx$  (10 pts)

3. Evaluate  $\int e^x \cos x dx$  (10 pts)

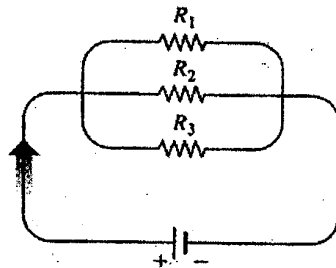
4. What is the area of the largest rectangle in the first quadrant with two sides on the axes and one vertex on the curve  $y = e^{-2x}$ . (15 pts)

5. In 3D space  $(x,y,z)$ , the plane  $P: x+y+z=1$  cuts the cylinder  $C: x^2+y^2=1$  in an ellipse  $E$ . Using Lagrange Multipliers to find the points on the ellipse  $E$  that lie closet to and farthest from the origin. (15 pts)

6. If resistors of  $R_1, R_2,$  and  $R_3$  ohms are connected in parallel to make an  $R$ -ohm resistor (see the figure), the value of  $R$  can be found from the equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Please find the value of  $\frac{\partial R}{\partial R_2}$  when  $R_1 = 30, R_2 = 20$  and  $R_3 = 60$  ohms. (10 pts)



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7. Write the expression of the tangent plane and calculate the normal vector of the surface  $f(x, y, z) = x^2 + y^2 + z^2 - 9 = 0$  at a point  $(1, 2, 4)$ . (10 pts)

8. Find the local extreme values of the following function:

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4 = 0. \text{ (15 pts)}$$